The submitted abstracts are not refereed at this stage and the authors are solely responsible for the claims made in their abstracts.

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Invited Talks

**Introducing Lexicographic Depth First Search**

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Recently there has been a great deal of interest in multisweep algorithms based on Lexicographic Breadth First Search (LBFS), introduced in a seminal paper by Rose, Tarjan and Lueker concerning the recognition of Chordal graphs. Study of the four-vertex characterization of LBFS has led to the discovery of a new graph search called Lexicographic Depth First Search (LDFS).

In this talk, we survey the many applications of LBFS and the characterizations of traditional searches that lead to the discover of LDFS. We then show how LDFS helps solve the long standing question of finding a minimum path cover of cocomparability graphs (graphs whose complement has a transitive orientation of its edges) that does not resort to determining the bump number of the poset associated with the complement graph. The talk concludes with open questions.

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**Some New and Old Results Regarding Room Squares and Related Resigns**

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In 1972 Wallis wrote the book on Room squares. Since that time there has been much work in the area of Room squares and related designs, but some problems are still unsolved. In this talk I will discuss some of these problems, in particular I will look at certain classes of Room squares, frames and 3-dimensional Howell designs. The talk will end with a piece of music which is based on a large set of Room squares.

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**A Greedy Sorting Algorithm**

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In sorting situations where the final destination of each item is known, it is natural to repeatedly choose items and place them where they belong, allowing the intervening items to shift by one to make room. However, it is not obvious that this algorithm necessarily terminates. We show that in fact the algorithm terminates after at most $2^n - 1$ steps in the worst case, and that there are super-exponentially many permutations for which this exact bound can be achieved. The proof involves a curious symmetrical binary representation.
Almost Resolvable Max Packings/Min Coverings Of K_n With 4-Cycles

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Let \((X, C)\) be a max packing/min covering of \(K_n\) with 4-cycles. An almost parallel class of \(C\) is a largest collection of vertex disjoint 4-cycles of \(C\). So the deficiency of an almost parallel class consists of 0, 1, 2, or 3 vertices. \((X, C)\) is said to be almost resolvable if \(C\) can be partitioned into almost parallel classes so that the remaining 4-cycles are vertex disjoint. For example, \((X, C)\) is an almost resolvable maximum packing of \(K_{11}\) with 4-cycles where:

\[
\begin{align*}
c_1 &= (4, 10, 8, 6)(5, 11, 9, 7); \\
c_2 &= (4, 9, 6, 11)(8, 7, 10, 5); \\
c_3 &= (1, 4, 5, 9)(2, 6, 3, 10); \\
c_4 &= (1, 6, 7, 11)(2, 4, 3, 8); \\
c_5 &= (1, 5, 6, 10)(2, 7, 3, 11); \\
c_6 &= (1, 7, 4, 8)(2, 5, 3, 9), \\
c_7 &= (8, 9, 10, 11),
\end{align*}
\]

This is an elementary talk (maybe too elementary) outlining a complete solution of this problem (including when \(n \equiv 1 (\text{mod} \ 8)\) a 4-cycle system).

Maximal Sets in Graph Decompositions

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I am very thankful to Wal for being one of the pioneers of research into the existence of sets of factors that are maximal in that none exist in the complement of the graph induced by the edges in the factors. The complement is taken in the natural families of complete graphs and complete multipartite graphs. This talk will include some historical perspectives on results in this area that involve Wal’s work, and some more recent results, one being hot off the press.

Hats

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In a game show, a team of \(n\) players competes for a shared prize of $1,000,000. Each contestant enters the studio blindfolded, and a hat is placed on his/her head. The hats are either black or red. The allocation of hat colors is independent and random, with each player having 50% chance of red (a fair coin is tossed for each player). When they are all hatted, the blindfolds are removed. A contestant cannot see his/her own hat, but can see all the others. No communication is allowed. Each contestant has to guess the color of his/her own hat (the contestant is also allowed to pass). They write down their answers – either red or black or pass – independently and simultaneously, so that none has any idea of the other players’ responses. If there is even one wrong answer, they lose. If they all pass, they lose. To win the money at least one player must guess the correct color, and no one can get it wrong.

The players are told the rules, and that the allocation of hat colors is independent and random (50%). They can then decide on a strategy. But remember, once the game starts there is no communication, and no player knows any other player’s response before making his/her own.

What is the team’s best strategy?
Given weights on the vertices of a graph (initially all weights equal 1), an acquisition move transfers weight to a vertex \(v\) from a neighbor \(u\) such that the weight on \(u\) is at most the weight on \(v\). The process ends when the vertices with positive weight form an independent set. The aim is to minimize the size of that final set. The full, partial, and fractional acquisition numbers are the minimum sizes of the final set when the amount of weight transferred in a move is required to be all weight on \(u\), any integer portion of that weight, or any portion of the weight, respectively. We discuss bounds on these parameters in various families of graphs and conditions for when they equal 1.

Finally, the game acquisition number is the result of optimal play in the variation where two players alternately make full acquisition moves, with one trying to minimize and the other trying to maximize the final set. We discuss game acquisition for complete bipartite graphs.

Individuals involved in this research include T. LeSaulnier, K. Milans, N. Prince, P. Wenger, L. Wiglesworth, and P. Worah.
Contributed Talks

On Representations of Graphs Mod n

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Let $G$ be a finite graph with vertices $\{v_1, v_2, \ldots, v_r\}$. A representation of $G$ is an assignment of distinct labels to the vertices such that the label $a_i$ assigned to vertex $v_i$ is in $\{0, 1, 2, \ldots, n-1\}$ and such that $\gcd(a_i - a_j, n) = 1$ if and only if $v_i$ and $v_j$ are adjacent. We call $\{a_1, a_2, \ldots, a_r\}$ a representation of $G$ modulo $n$. The representation number of a graph $G$, $\text{rep}(G)$, is the smallest $n$ such that $G$ has a representation modulo $n$. In this talk we will present the representation numbers of a family of complete graphs minus a disjoint union of paths.

On the Oberwolfach Problem

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Let $F$ be a 2-regular graph of order $n$, let $K_n$ be the complete graph of order $n$, and for even $n$ let $K_n - I$ be the complete graph of order $n$ with the edges of a perfect matching removed. When $n$ is odd the Oberwolfach problem OP($F$) asks for a 2-factorisation of $K_n$ into copies of $F$, and when $n$ is even it asks for a 2-factorisation of $K_n - I$ into copies of $F$. In this talk I will outline a construction which settles the Oberwolfach problem for infinitely many odd and even values of $n$.

Characterizing the Intersection Graphs of Paths in Trees Using PR-Trees

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A graph, $G = (V=\{v_1, \ldots, v_k\}, E)$, is an intersection graph of paths in a tree (a.k.a. VPT graph a.k.a. UV graph a.k.a. undirected path graph) iff there exists a collection of paths $\{P_1, \ldots, P_k\}$ whose union forms a tree, and there is an edge between $v_i$ and $v_j$ if and only if $P_i$ and $P_j$ have at least one vertex in common. Additionally, by Gavril [1], a graph, $G$, is VPT iff there exists a tree, $T$, whose vertices correspond to the maximal cliques of $G$, such that for every vertex, $v$, in $G$, the set of incident maximal cliques of $G$ forms a path (i.e. each path $P_i$ consists of the maximal cliques incident with $v_i$). We refer to such a tree, $T$, as a VPT representation of $G$.

In this talk we present a data structure (PR-trees) to represent the set of VPT representations of a graph $G$. Furthermore, we demonstrate a polytime algorithm to construct a PR-tree from a given graph (note: when there are no VPT representations an empty PR-tree is produced).
A Non-Existence Result and Large Sets for SB Designs

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For $n > 4$, a strict $(n-2)$-SB$(n,n-1)$ does not exist. In this note, we prove a general result that for $3 \leq t \leq (n-3)$, a strict $t$-SB$(n,n-1)$ does not exist. We modify the definition of Large sets of balanced incomplete block designs appropriately for SB designs. An easy construction of such large sets for block size two for all values of $n$ is given as well as an example of a large set of an SB triple system for $n = 4$.

On the Edge-graceful Indices of the $L$-product of $(p, p+1)$-Graphs and $K_2$

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Let $G$ be a $(p,q)$-graph and $k > 0$. A graph $G$ is said to be $k$-edge graceful if the edges can be labeled by $k, k+1, \ldots, k+q-1$ so that the vertex sums are distinct, $(\text{mod } p)$. We denote the set of all $k$ such that $G$ is $k$-edge graceful by egI$(G)$. The set is called the edge-graceful spectrum of $G$. In this paper the problem of what sets of natural numbers are the edge-graceful spectra of three types of $(p,p+1)$-graphs, namely the $L$-product cycles with one chord, dumbbell graphs and one point union of cycles with $K_2$ graph are studied.

On Some Combinatorial Arrays

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An array $T$ with $m$ rows (constraints), $N$ columns (runs, treatment-combinations), and $s$ levels is merely a matrix $T$ $(m \times N)$ with elements from a set $S = \{0,1,2,\ldots,s-1\}$. These arrays assume great importance in combinatorics and statistical design of experiments, when one imposes some combinatorial structure on them. In this paper, we restrict ourselves to arrays with two elements 0 and 1. An array $T$ is called an orthogonal array (O-array) of strength $t$ $(0 \leq t \leq m)$ if in each $(t \times N)$ submatrix $T^*$ of $T$, every $(t \times 1)$ vector $\alpha$ with $i$ $(0 \leq i \leq t)$ ones in it appears with the same frequency $\lambda$ (say). Here, $\lambda$ is called the index set of the O-array. Clearly, $N = \lambda \cdot 2^t$. These arrays have been extensively used in design of experiments, in coding and information theory, etc. Balanced arrays (B-arrays) are generalizations of O-arrays in the sense that the combinatorial structure imposed on O-arrays is somewhat weakened, where every vector $\alpha$ of weight $i$ $(0 \leq i \leq t)$ appears with the same frequency $\lambda_i$ (say). Clearly if $\lambda_i = \lambda$ for each $i$, one gets an O-array. In this paper, we will present some results on the existence of some of these combinatorial arrays.
On the Product-Cordial Index Sets of $C_m \times P_n$

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Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$, and let $<Z_2, \oplus, \odot>$ be the field of two elements where $Z_2 = \{0, 1\}$. A labeling $f : V(G) \to Z_2$ induces an edge labeling $f^* : E(G) \to Z_2$ defined by $f^*(xy) = f(x) \oplus f(y)$, for each edge $xy \in E(G)$. For $i \in Z_2$, let $e_f(i) = \text{card}\{v \in V(G) : f(v) = i\}$ and $e_{f^*}(i) = \text{card}\{e \in E(G) : f^*(e) = i\}$. A labeling $f$ of a graph $G$ is said to be friendly if $|e_f(0) - e_f(1)| \leq 1$. If $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ then $G$ is said to be product cordial. The product-cordial index set of the graph $G$, $PI(G)$, is defined as $\{|e_f(0) e_{f^*}(1) : \text{the vertex labeling } f \text{ is friendly}\}$. Similarly, a labeling $g : V(G) \to Z_2$ induces an edge labeling $g^* : E(G) \to Z_2$ defined by $g^*(xy) = g(x) \oplus g(y)$, for each edge $xy \in E(G)$. For $i \in Z_2$, let $v_g(i) = \text{card}\{v \in V(G) : g(v) = i\}$ and $e_{g^*}(i) = \text{card}\{e \in E(G) : g^*(e) = i\}$. A labeling $g$ of a graph $G$ is said to be friendly if $|v_g(0) - v_g(1)| \leq 1$. If $|e_{g^*}(0) - e_{g^*}(1)| \leq 1$ then $G$ is said to be cordial. The friendly index set of the graph $G$, $FI(G)$, is defined as $\{|e_{g^*}(0) V e_{g^*}(1) : \text{the vertex labeling } g \text{ is friendly}\}$. In this paper we show that all cylinder graph $C_3 \times P_n$ are not product-cordial and multilayer $W_4$ wheels are product-cordial.

On Edge-balance Index Sets of Cubic Trees

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Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$, and let $Z_2 = \{0, 1\}$. A labeling $f : E(G) \to Z_2$ of a graph $G$ is said to be edge-friendly if $|e_f(0) - e_f(1)| \leq 1$. An edge-friendly labeling $f$ induces a partial vertex labeling $f^+ : V(G) \to Z_2$ defined by $f^+(x) = 0$ if the number of edges labeled by 0 incident on $x$ is more than the number of edges labeled by 1 incident on $x$. Similarly, $f^+(x) = 1$ if the number of edges labeled by 1 incident on $x$ is more than the number of edges labeled by 0 incident on $x$. $f^+(x)$ is not defined if the number of edges labeled by 0 incident on $x$ is equal to the number of edges labeled by 0 incident on $x$. A tree is called cubic if all internal vertices are of degree 3. In this paper, exact values of the edge-balance index sets of cubic trees are obtained, all of them form arithmetic progressions.

On The Mod(2)-Edge-Magic Graphs

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Let $G$ be a $(p, q)$-graph in which the edges are labeled by $1, 2, \cdots, q$. The vertex sum for a vertex $v$ is the sum of the labels of the incident edges at $v$. If the vertex sums are constant, mod $k$, where $k > 2$, then $G$ is said to be Mod($k$)-edge-magic. In this paper we investigate graphs which are Mod($k$)-edge-magic. When $k = p$, the Mod($k$)-edge-magic graph is the edge-magic graph which was introduced by the third author, Eric Seah and S.K. Tan in 1992. We characterize trees and unicyclic graphs which are Mod(2)-edge-magic. In particular, no trees of odd orders are Mod(2)-EM.
On decomposing complete graphs into Hamilton cycles and $n$-cycles

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Let $n \geq 3$ be an integer and let $m \geq 2n+1$ be odd. It is known that if $m \equiv 1 \pmod{2n}$, then there exists a cyclic $C_n$-decomposition of $K_m$. Let $k$ be a positive integer and let $m = 2nx + 2k - 1$. We show that if $n$ is even or if $1 \leq k \leq n$ when $n$ is odd and $(n,k) \neq (5,3)$, then there exists a cyclic $C_n$-decomposition of $K_{2nx+2k-1} - H$, where $H$ is a $(2k-2)$-regular spanning graph that can be factored into Hamilton cycles.

Matrices and Their Kirchhoff Graphs

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Given a matrix with integer or rational elements, this talk will introduce the concept of a Kirchhoff or fundamental graph for this matrix. A Kirchhoff graph represents the fundamental theorem of linear algebra for the matrix. The construction of these graphs and some of their properties will be discussed. The final portion of this talk describes the connection between these graphs and chemical reaction networks which motivates their definition.

Total Coloring of Cactus Graphs

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In this paper, we present new results about coloring graphs. We generalize the notion of proper vertex coloring presenting the concept of range-coloring of order $k$. The usual vertex coloring of $G$ is a range-coloring of order 1; we prove the equivalence of the range-coloring of order $\Delta(G)$ and the distant-2 coloring. The relation between range-coloring of order $k$ and total coloring is presented: we show that for any graph $G$ that has a range-coloring of order $\Delta(G)$ with $t$ colors, there is a total coloring of $G$ that uses $(t+1)$ colors. This result provides a framework to prove that some families of graphs satisfy the Vizing conjecture for total coloring. We exemplify with the family of cactus graphs.
**Distance Magic Graphs and Tournament Scheduling**

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A graph $G$ with the vertex set $V(G)$, edge set $E(G)$ and $|V(G)| = n$ is called **distance magic** if there exists an injective mapping $\mu : V \to \{1, 2, \ldots, n\}$ such that the **weight** of each vertex $x$, defined as

$$w(x) = \sum_{xy \in E(G)} \mu(y),$$

is equal to the same constant $\mu_0$, called the **magic constant**. The labeling is called a **distance magic labeling**. In some papers, $\mu$ is also called a 1-vertex-magic vertex labeling.

We present constructions of **fair** and **handicap** incomplete round robin tournaments based on distance magic graphs. If time permits, we may also present a construction of distance magic graphs arising from arbitrary regular graphs based on an application of Kotzig arrays and present a solution of a problem posed by Shafiq, Ali and Simanjuntak.

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**On the Edge Balance Index Sets of Flux Capacitor and L-Product of Stars by Cycle Graphs**

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Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$, and let $\mathbb{Z}_2 = \{0, 1\}$. A labeling $f : E(G) \to \mathbb{Z}_2$ of a graph $G$ is said to be edge-friendly if $|e_f(0) - e_f(1)| \leq 1$. An edge-friendly labeling $f$ induces a partial vertex labeling $f^+ : V(G) \to \mathbb{Z}_2$ defined by $f^+(x) = 0$ if the number of edges labeled by 0 incident on $x$ is more than the number of edges labeled by 1 incident on $x$. Similarly, $f^+(x) = 1$ if the number of edges labeled by 1 incident on $x$ is more than the number of edges labeled by 0 incident on $x$. $f^+(x)$ is not defined if the number of edges labeled by 1 incident on $x$ is equal to the number of edges labeled by 0 incident on $x$. For $i \in \mathbb{Z}_2$, let $v_f(i) = \text{card}\{v \in V(G) : f^+(v) = i\}$ and $e_f(i) = \text{card}\{e \in E(G) : f(e) = i\}$. The edge-balance index set of the graph $G$, $\text{EBI}(G)$, is defined as $\{|v_f(0) - v_f(1)| : \text{the edge labeling } f \text{ is edge-friendly}\}$. The edge-balance index sets of Flux Capacitor graphs and L-Product with Stars by Cycles graphs are presented in this paper.
Partial Covering Arrays and a Generalized Erdős-Ko-Rado Property

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The classical Erdős-Ko-Rado theorem states that if \( k \leq \lfloor n/2 \rfloor \), then the largest family of pairwise intersecting \( k \)-subsets of \( [n] = \{0, 1, \ldots, n\} \) is of size \( \binom{n-1}{k-1} \). A family of \( k \) subsets satisfying this pairwise intersecting property is called an EKR family. We generalize the EKR property and provide asymptotic lower bounds on the size of the largest family \( \mathcal{A} \) of \( k \)-subsets of \( [n] \) that satisfies the following property: For each \( A, B, C \in \mathcal{A} \), each of the four sets \( A \cap B \cap C \), \( A \cap B \cap C^c \), \( A \cap B^c \cap C \), \( A^c \cap B \cap C \) are non-empty. This generalized EKR (GEKR) property is motivated, generalizations are suggested, and a comparison is made with fixed weight 3-covering arrays. Our techniques are probabilistic, and reminiscent of those used by Godbole et al (1996) and in the doctoral work of Roux, as reported in the seminal paper of Sloan (1993).

Negative Cost Cycle Detection Problem in Undirected Graphs

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In this presentation, we explore the design of algorithms for the problem of checking whether a weighted undirected graph contains a negative cost cycle (UNCCD). The mainly algorithm uses a \( T \)-join approach, which runs in \( O(n^3) \) time. We also show that if edge weights are restricted to be integers in \( [-K, K] \), where \( K \) is a constant, then the problem can be solve in \( O(n^{2.75} \cdot \log n) \) time.

A Lexicographical Approach to SAT

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Lexicographical compatibility matrix is a Boolean box matrix which encodes a given CNF in the lexicographical contradictions between clauses. The given CNF is satisfiable iff there is a special pattern in the matrix - a solution grid. The solution grids are in one-to-one relation with the implicants in DNF of the given CNF. Disjunction of all solution grids is called a general solution of SAT. Search for solution grids can be reduced to an asymmetric polynomial size linear system. This work researches another approach - a dynamic programming procedure which is called depletion of the compatibility matrix. Formally, depletion is just inversion in the compatibility matrix some of its true-elements. The depletion may be organized in such a way that its result will be the general solution of SAT.
On Maximal $k$-Limited Packings

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A $k$-limited packing $P$ is a set of vertices in a graph with the property that the closed neighborhood of any vertex in the graph contains at most $k$ members of $P$. This could model, for instance, the desire to have a limited number of obnoxious facilities in one’s backyard. We consider the following two player game based on this concept. The players alternate choosing a vertex in a graph. The only restriction is that at most $k$ vertices can be selected in the closed neighborhood of any vertex. Some preliminary observations from this investigation will be outlined.

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Sarvate-Beam Triple Systems for $v = 5$ and $v = 6$

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A Sarvate-Beam Triple System $SB(v, 3)$ is a set $V$ of $v$ elements and a collection of 3-subsets of $V$ such that each distinct pair of elements in $V$ occurs $i$ times for every $i$ in the list $1, 2, \ldots, (v^2)$. In this paper, we completely enumerate all Sarvate-Beam Triple Systems for $v = 5$ and $v = 6$. (In the case $v = 5$, we extend a previous result of R. Stanton.)

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Weak Colourings of Cycle Systems

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An $m$-cycle system of order $v$ is a decomposition of the complete graph on $v$ vertices into cycles of length $m$. A weak colouring of an $m$-cycle system is a colouring of the vertices of the system such that no cycle has all of its vertices receive the same colour, and the weak chromatic number of a system is the smallest number of colours with which the system can be weakly coloured. This talk will deal with the problem of constructing $m$-cycle systems with specified orders and weak chromatic numbers. Along the way we mention a new result on embedding partial odd-cycle systems.
On Graphs Having Edge-magic Index Number Two

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A \((p, q)\)-graph \(G\) in which the edges are labeled 1, 2, 3, \cdots, \(q\) so that the vertex sums are constant, is called supermagic. If the vertex sum \(\pmod p\) is a constant, then \(G\) is called edge-magic. A necessary condition of edge-magicness is \(p\) divides \(q(q + 1)\). Lee, Seah and Tan showed that for any graph \(G\) there is an integer \(k\) such that the \(k\)-fold graph \(G[k]\) is edge-magic. The least such integer \(k\) is called the edge-magic index of \(G\). We characterize some graphs whose edge-magic indices are two.

Cycle Designs With Loops

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We consider graph designs which decompose \(K_v(\lambda, j)\), the complete multi-graph on \(v\) points with \(\lambda\) multiple edges for each pair of points and \(j\) loops at each vertex, into ordered blocks \((a_1, a_2, \ldots, a_{k-1}, a_1)\) for \(k = 3, 4\), and have partial results for \(k = 5\). Each block is the subgraph which contains the unordered edges \(\{a_i, a_j\}\), for each pair of consecutive edges in the ordered list and contains also the loop at the vertex \(a_1\). We have complete results for \(k = 3, 4\) and partial results for \(k = 5\). Each block is the subgraph which contains the unordered edges \(\{a_i, a_j\}\), for each pair of consecutive edges in the ordered list and contains also the loop at the vertex \(a_1\).

Star Avoiding Ramsey Numbers

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For graph Ramsey number \(R(G, H) = r\) what is the largest star such that we can leave its edges uncolored in two coloring the complete graph on \(r\) vertices and still force a red \(G\) or blue \(H\)? For \(G\) and \(H\) complete this is not interesting, all edges must be colored. We determine values for some special cases of \(G\) and \(H\) (paths, cycles)where \(R(G, H)\) is known.

On the Irreducible No-Hole \(L(2, 1)\) Labeling of Graphs

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Let \(G\) be a graph. A labeling \(f : V(G) \rightarrow \{0, 1, \ldots, k\}\) of \(G\) is an \(L(2, 1)\) labeling if \(|f(u) - f(v)| \geq 2\) when \(u\) and \(v\) are adjacent in \(G\), and \(|f(u) - f(v)| \geq 1\) when \(u\) and \(v\) are at distance two in \(G\). An \(L(2, 1)\) labeling \(f\) is a no-hole \(L(2, 1)\) labeling if \(f\) is onto. An \(L(2, 1)\) labeling is irreducible if reduction of any label to a smaller label violates the conditions of \(L(2, 1)\) labeling.

In this talk we will discuss some results regarding irreducible no-hole \(L(2, 1)\) labelings of different graphs.
Single Block Attacks and Statistical Tests on CubeHash

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This paper describes a second preimage attack on the CubeHash cryptographic one-way hash function. The attack finds a second preimage in less time than brute force search for these CubeHash variants: CubeHash \( r/b-224 \) for \( b > 100 \); CubeHash \( r/b-256 \) for \( b > 96 \); CubeHash \( r/b-384 \) for \( b > 80 \); and CubeHash \( r/b-512 \) for \( b > 64 \). However, the attack does not break the CubeHash variants recommended for SHA-3. The attack requires minimal memory and can be performed in a massively parallel fashion. This paper also describes several statistical randomness tests on CubeHash. The tests were unable to disprove the hypothesis that CubeHash behaves as a random mapping. These results support CubeHash’s viability as a secure cryptographic hash function.

Vertex-Magic Edge Labeling Games on Graphs with Cycles

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Given a graph \( G \), let \( E \) be the number of edges in \( G \). A vertex-magic edge labeling of \( G \), defined by Wallis in 2001, is a one-to-one mapping from the set of edges onto the set \( \{1, 2, \ldots, E\} \) with the property that at any vertex the sum of the labels of all the edges incident to that vertex is the same constant. In 2003, Hartnell and Rall introduced a two player game based on these labelings, and proved some nice results about winning strategies on graphs that contain vertices of degree one. In this talk we discuss results about winning strategies for certain graphs with cycles where the minimum degree is two.

On Balance Index Sets of Generalized Wheel Graphs

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Let \( G \) be a graph with vertex set \( V(G) \) and edge set \( E(G) \), and let \( A = \{0, 1\} \). A labeling \( f : V(G) \to A \) induces an edge partial labeling \( f^* : E(G) \to A \) defined by \( f^*(xy) = f(x) \), if and only if \( f(x) = f(y) \) for edge \( xy \in E(G) \). For \( i \in A \), let \( v_f(i) = \text{card}\{v \in V(G) : f(v) = i\} \) and \( e_f(i) = \text{card}\{e \in E(G) : f^*(e) = i\} \). \( G \) is said to be a balanced graph or \( G \) is balanced, if there is a vertex labeling \( f \) of \( G \) that satisfies the following conditions:

(1) \(|v_f(0) - v_f(1)| \leq 1 \) and,
(2) \(|e_f(0) - e_f(1)| \leq 1 \).

The balance index set of the graph \( G \), \( BI(G) \), is defined as \( \{v_f(0) - e_f(1) : \text{the vertex labeling } f \text{ is friendly}\} \). The Zykov sum of \( C_n \) and \( N_m \) is called the generalized wheel, and is denoted by \( GW(n, m) \). In this paper we determine balance index sets of generalized wheels for \( m = 2, 3, 4, 5 \).
On Friendly Index Sets of \((p, p + 1)\)-Graphs

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Let \(G\) be a graph with vertex set \(V(G)\) and edge set \(E(G)\). A vertex labeling \(f : V(G) \rightarrow \mathbb{Z}_2\) induces an edge labeling \(f^* : E(G) \rightarrow \mathbb{Z}_2\) defined by \(f^*(xy) = f(x) + f(y)\) for each edge \(xy \in E(G)\). For each \(i \in \mathbb{Z}_2\), define \(v_f(i) = |\{v \in V(G) : f(v) = i\}|\) and \(e_f(i) = |\{e \in E(G) : f^*(e) = i\}|\). The friendly index set of the graph \(G\), denoted \(\text{FI}(G)\), is defined as \(|e_f(0) - e_f(1)|\). In this paper, we determine the friendly index sets of some \((p, p + 1)\)-graphs. Many of which form arithmetic progressions. Those that are not miss only the second to the last term of the progressions.

Computing the Folkman Number \(F_v(2, 2, 2, 2; 4)\)

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For a graph \(G\), the expression \(G \xrightarrow{r} (a_1, \ldots, a_r)\) means that for any \(r\)-coloring of the vertices of \(G\) there exists a monochromatic \(a_i\)-clique in \(G\) for some color \(i \in \{1, \ldots, r\}\). The vertex Folkman numbers are defined as \(F_v(a_1, \ldots, a_r; q) = \min\{|V(G) : G \xrightarrow{r} (a_1, \ldots, a_r) \text{ and } K_q \not\subseteq G\}|\)

Of these, the only Folkman number of the form \(F_v(2, 2, 2, 2, 2; 4)\) which has remained unknown up to this time is \(F_v(2, 2, 2, 2; 4)\). We show here that \(F_v(2, 2, 2, 2; 4) = 16\), which is equivalent to saying that the smallest 6-chromatic, \(K_4\)-free graph has 16 vertices. We also show that the sole witnesses of the upper bound \(F_v(2, 2, 2, 2; 4) \leq 16\) are the two Ramsey \((4,4)\)-graphs on 16 vertices.

Partition Types

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For a graph \(G\) having chromatic number \(k\), an equivalence relation is defined on the set \(X\) consisting of all proper vertex \(k\)-colorings of \(G\). This leads naturally to an equivalence relation on the set \(\mathcal{P}\) consisting of all partitions of \(V(G)\) into \(k\) independent subsets of color classes. The notion of a partition type arises and the algebra of types is investigated. Visual assistance is provided by Mathematica.
On Balance Index Sets of Some Bi-regular and Tri-regular Graphs

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Let $G$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$, and let $A = \{0, 1\}$. A labeling $f : V(G) \rightarrow A$ induces an edge partial labeling $f^*: E(G) \rightarrow A$ defined by $f^*(xy) = f(x)$ if and only if $f(x) = f(y)$ for each edge $xy \in E(G)$. For $i \in A$, let $v_f(i) = \text{card}\{v \in V(G) : f(v) = i\}$ and $e_f(i) = \text{card}\{e \in E(G) : f^*(e) = i\}$. A labeling $f$ of a graph $G$ is said to be balanced. The balance index set of a class of bi-regular graphs and a class of tri-regular graphs are investigated.

On cordialness of $k^{th}$ power of graphs

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Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$. A labeling $f : V(G) \rightarrow \{0, 1\}$ induces an edge labeling $f^* : E(G) \rightarrow \{0, 1\}$, defined by $f^*(xy) = |f(x) - f(y)|$, for each edge $xy \in E(G)$. For $i \in \{0, 1\}$, let $n_i(f) = \{v \in V(G) : f(v) = i\}$ and $m_i(f) = \{e \in E(G) : f^*(e) = i\}$. A labeling $f$ of a graph $G$ is cordial if $|n_0(f) - n_1(f)| \leq 1$ and $|m_0(f) - m_1(f)| \leq 1$. In this paper we show that under what conditions on $G$, the $k$-th power $G^k$ of $G$ is cordial. Finally, we discuss the cordiality of $C_n^k$.

New Bounds on Some Ramsey Numbers

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We derive a new upper bound of 26 for the Ramsey number $R(K_5 - P_3, K_5)$, improving on the previous upper bound of 28. This leaves $25 \leq R(K_5 - P_3, K_5) \leq 26$, improving on one of the three remaining open cases in Hendry’s tables, which listed Ramsey numbers for pairs of graphs $(G, H)$ with $G$ and $H$ having five vertices.

We also show, with the help of a computer, that $R(B_2, B_6) = 17$ and $R(B_2, B_7) = 18$ by full enumeration of $(B_2, B_6)$-good graphs and $(B_2, B_7)$-good graphs, where $B_n$ is the book graph with $n$ triangular pages.
Hamiltonian Properties in Cartesian Product

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Ron Gould in [Problem 6, Graphs and Combinatorics, (2003)] raise a research problem to find natural conditions to assure the product of two graphs to be hamiltonian. We prove that the following results:

(i) Let $G$ be a hamiltonian graph and let $T$ be a tree, then the Cartesian product $G \times H$ is hamiltonian if and only if the maximum degree of $T$ satisfies $\Delta(T) \geq |V(G)|$.

(ii) Let $G$ be a hamiltonian graph and let $T$ be a tree, then the Cartesian product $G \times H$ is traceable if and only if either the maximum degree of $T$ satisfies $\Delta(T) \geq |V(G)|$ or $\Delta(T) = |V(G)| + 1$ and any subdivisions of $K_{1,3}(V(G))$ is not a subgraph of $T$. (Where $K_{1,3}(n)$ is the graph identifying every degree one vertex of $K_{1,3}$ with the center of a $K_{1,n}$.)

Contractible Graphs with Respect to Mod $(2p+1)$-Orientations

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An orientation of an undirected graph $G$ is a mod $(2p+1)$-orientation if under this orientation, the net out-degree at every vertex is congruence to zero mod $2p+1$. A graph $H$ is mod $(2p+1)$-contractible if for any graph $G$ that contains $H$ as a subgraph, the contraction $G/H$ has a mod $(2p+1)$-orientation if and only if $G$ has a mod $(2p+1)$-orientation (thus every mod $(2p+1)$-contractible graph has a mod $(2p+1)$-orientation). Jaeger in 1984 conjectured that every $(4p)$-edge-connected graph has a mod $(2p+1)$-orientation. It has also been conjectured that every $(4p+1)$-edge-connected graph is mod $(2p+1)$-contractible. In this paper, we investigate graphs that are mod $(2p+1)$-contractible, and as applications, we prove that a complete graph $K_m$ is $(2p+1)$-contractible if and only if $m \geq 4p+1$; that every $(4p-1)$-edge-connected $K_4$-minor free graph is mod $(2p+1)$-contractible, which is best possible in the sense that there are infinitely many $(4p-2)$-edge-connected $K_4$-minor free graphs that are not mod $(2p+1)$-contractible; and that every $(4p)$-connected chordal graph is mod $(2p+1)$-contractible.

On Almost-Edge-Graceful Trees

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Let $G$ be a $(p,q)$-graph in which the edges are labeled by $1, 2, \cdots, q$. The vertex sum for a vertex $v$ is the sum of the labels of the incident edges at $v$. If the vertex sums are constant, mod $p$, then $G$ is said to be edge-magic. The edge-magic graphs were introduced by Lee, Eric Seah and S.K. Tan in 1992. If the vertex set of $G$ can be decomposed by $V_1, V_2, \cdots, V_k$ where $p-1 > k > 2$, such that the vertex sums of vertices in $V_i$ are constant, mod $p$, then we say $G$ is split$(k)$-edge-magic. In particular split $(p-1)$-edge magic graph is called almost edge-graceful graph. In this paper we investigate trees which are almost-edge-graceful. We conjecture that all trees of even orders are almost edge-graceful.
On (p, p + 1)-Graphs whose Edge-magic Indices are 3

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A (p, q)-graph $G$ in which the edges are labeled 1, 2, 3, · · · , q so that the vertex sums are constant, is called supermagic. If the vertex sum (mod $p$) is a constant, then $G$ is called edge-magic. A necessary condition of edge-magicness is $p$ divides $q(q + 1)$. Lee, Seah and Tan showed that for any graph $G$ there is an integer $k$ such that the $k$-fold graph $G[k]$ is edge-magic. The least such integer $k$ is called the edge-magic index of $G$. We complete characterize $(p, p + 1)$-graphs whose edge-magic indices are three.

On Balance Index Sets of Corona of Regular Graphs and Regular Graphs

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Let $G$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$, and let $A = \{0, 1\}$. A labeling $f : V(G) \to A$ induces an edge partial labeling $f^* : E(G) \to A$ defined by $f^*(xy) = f(x)$ if and only if $f(x) = f(y)$ for each edge $xy \in E(G)$. For $i \in A$, let $v_f(i) = \text{card}\{v \in V(G) : f(v) = i\}$ and $e_f(i) = \text{card}\{e \in E(G) : f^*(e) = i\}$. A labeling $f$ of a graph $G$ is said to be friendly if $|v_f(0) - v_f(1)| \leq 1$. If $|e_f(0) - e_f(1)| \leq 1$ then $G$ is said to be balanced. The balance index set of the graph $G$, $BI(G)$, is defined as $\{ |e_f(0) - e_f(1)| :$ the vertex labeling $f$ is friendly.}. The corona of two graphs $G$ and $H$, written as $G \circ H$, is the graph obtained by taking one copy of $G$ and $|V(G)|$ copies of $H$, and then joining the $i$th vertex of $G$ to every vertex in the $i$th copy of $H$. For a connected graph $a$-regular graph $G$ and any $b$-regular graph $H$ we provide complete information about the balance index set of $G \circ H$.

An Improved Kernel Size for Rotation Distance in Binary Trees

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Kernelization is one technique for studying intractable problems. By pre-processing a problem instance, and converting it into an equivalent smaller instance, one can substantially lower the time needed to obtain a solution. In this talk we consider the size of the kernel for the problem of computing the rotation distance between a pair of binary trees, or, equivalently, computing the diagonal flip distance between a pair of convex polygons. We show an improved bound of $2k$ on the size of the kernel.
On Balance Index Sets of Cylinder Graphs and Grid Graphs

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Let $G$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$, and let $A = \{0, 1\}$. A labeling $f : V(G) \to A$ induces an edge partial labeling $f^* : E(G) \to A$ defined by $f^*(xy) = f(x)$ if and only if $f(x) = f(y)$ for each edge $xy \in E(G)$. For $i \in A$, let $v_f(i) = \text{card}\{v \in V(G) : f(v) = i\}$ and $e_f^*(i) = \text{card}\{e \in E(G) : f^*(e) = i\}$. A labeling $f$ of a graph $G$ is said to be friendly if $|v_f(0) - v_f(1)| \leq 1$. If $|e_f^*(0) - e_f^*(1)| \leq 1$ then $G$ is said to be balanced. The balance index set of the graph $G$, $\text{BI}(G)$, is defined as $\{|e_f^*(0) - e_f^*(1)| :$ the vertex labeling $f$ is friendly.$\}$. In this paper, the balance index set of cylinder graphs are investigated. In particular, we show that all the cylinder graphs and grid graphs are balanced.

On the Chromatic Number of $H$-Free Graphs of Large Minimum Degree

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The problem of determining the chromatic number of $H$-free graphs has been well studied, with particular attention to $K_r$-free graphs with large minimum degree. Recent progress has been made for triangle-free graphs on $n$ vertices with minimum degree larger than $n/3$. In this talk, we will discuss the family of three-colorable graphs $\mathcal{F}$, such that if $H \in \mathcal{F}$, there exists a constant $C < 1/2$ such that the chromatic number of any $H$-free graph $G$ with $\delta(G) > (C + \alpha)n$ can be bounded above by a function of $\alpha$ and $H$.

Pairs of Decompositions Into Lists of Cycles

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Suppose $M = m_1, m_2, \ldots, m_s$ and $N = n_1, n_2, \ldots, n_t$ are arbitrary lists of positive integers. In this talk I will present necessary and sufficient conditions on $M$ and $N$ for the existence of a simple graph whose edge-set can be partitioned into $s$ cycles of lengths $m_1, m_2, \ldots, m_s$ and also into $t$ cycles of lengths $n_1, n_2, \ldots, n_t$.

Some Families of Fixed Points for the Eccentric Digraph Operator

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The eccentric digraph $ED(G)$ of a digraph $G$ has the same vertex set as $G$, and has a directed edge from $u$ to $v$ if and only if the distance from $u$ to $v$ in $G$ is $e(u)$. A digraph is a fixed point under the $ED$ operator if $ED(G) \cong G$. This talk looks at several examples of fixed points including a discussion of cycle products.
On \((n, k, \lambda)\)-Ovals and \((n, k, \lambda)\)-Cyclic Difference Sets, Multisets, and Related Topics

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Each fixed integer \( n \geq 2 \) has associated with it \( \left\lceil \frac{\pi}{n} \right\rceil \) rhombs, \( \rho_1, \rho_2, \ldots, \rho_{\left\lceil \frac{\pi}{n} \right\rceil} \). Rhomb \( \rho_n \) is a parallelogram with smaller face angle equal to \( \frac{\pi}{n} \) radians. An Oval is an equilateral centro-symmetric convex polygon, each of whose turning angles equals \( \ell \times \frac{\pi}{n} \) for some positive integer \( \ell \). It is tiled by the rhombs \( \rho_1, \rho_2, \ldots, \rho_{\left\lceil \frac{\pi}{n} \right\rceil} \). An Oval with \( 2k \) sides is called a ‘\((n, k)\)-Oval’; it is described by its values of \( n \) and \( k \) and by its Turning Angle Index Sequence (‘TAIS’), a list of the turning angle indices for any consecutive set of \( k \) vertices. We are interested in \((n, k)\)-Ovals for which each rhomb is used \( \lambda \) times, we call these magic \((n, k, \lambda)\)-Ovals. They exist just when a \((n, k, \lambda)\)-CDS, (cyclic difference set), exists. We also consider pseudo-CDS, multipliers of CDS, and complements of Ovals.

Product Cordial Sets of Grids \( P_m \times P_n \)

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Let \( G \) be a graph, and \( f : V(G) \to \mathbb{Z}_2 \) be a binary vertex coloring (labeling) of \( G \). For \( i \in \mathbb{Z}_2 \), let \( v_f(i) = |f^{-1}(i)| \). The coloring \( f \) is said to be friendly if \( |v_f(1) - v_f(0)| \leq 1 \). That is, the number of vertices colored 0 is almost the same as the number of vertices colored 1.

Any friendly vertex coloring \( f : V(G) \to \mathbb{Z}_2 \) induces an edge labeling \( f^* : E(G) \to \mathbb{Z}_2 \) defined by \( f^*(xy) = f(x)f(y) \forall xy \in E(G) \). For \( i \in \mathbb{Z}_2 \), let \( e_f(i) = |f^*^{-1}(i)| \) be the number of edges of \( G \) that are labeled \( i \). The number \( pc(f) = |v_f(1) - v_f(0)| \) is called the product-cordial index (or pc-index) of \( f \). The product-cordial set (or pc-set) of the graph \( G \), denoted by \( PC(G) \), is defined by

\[
PC(G) = \{ pc(f) : f \text{ is a friendly vertex coloring of } G \}.
\]

In this talk we present the product-cordial sets of grids \( P_m \times P_n \).

Intermediate Minimal \( k \)-rankings of Graphs

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Given a graph \( G \), a function \( f : V(G) \to \{1, 2, \ldots, k\} \) is a \( k \)-ranking of \( G \) if \( f(u) < f(v) \) implies every \( u - v \) path contains a vertex \( w \) such that \( f(u) > f(v) \). A \( k \)-ranking is minimal if the reduction of any label greater than 1 violates the described ranking property. The rank number of a graph, denoted \( \chi_r(G) \), is the minimum \( k \) such that \( G \) has a minimal \( k \)-ranking. The arank number of a graph, denoted \( \psi_r(G) \), is the maximum \( k \) such that \( G \) has a minimal \( k \)-ranking. It was asked by Laskar, Pillone, Eyabi, and Jacob if there is a family of graphs where minimal \( k \)-rankings exist for all \( \chi_r(G) \leq k \leq \psi_r(G) \). We give an affirmative response to their question showing that all intermediate minimal \( k \)-rankings exist for all paths, cycles, and \( K_{n_1, n_2, \ldots, n_p} \) where \( n_{i+1} = n_i - 1 \) for all \( 1 \leq i \leq p - 1 \).
The Local Metric Dimension of a Graph

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For an ordered set $W = \{w_1, w_2, \ldots, w_k\}$ of $k$ distinct vertices in a nontrivial connected graph $G$, the metric code of a vertex $v$ of $G$ with respect to $W$ is the $k$-vector

$$\text{code}(v) = (d(v, w_1), d(v, w_2), \ldots, d(v, w_k))$$

where $d(v, w_i)$ is the distance between $v$ and $w_i$ for $1 \leq i \leq k$. The set $W$ is a local metric set of $G$ if $\text{code}(u) \neq \text{code}(v)$ for every pair $u, v$ of adjacent vertices of $G$. The minimum positive integer $k$ for which $G$ has a local metric $k$-set is the local metric dimension of $G$. Results concerning these concepts are presented.

On the Edge-balance Index Sets of Centipede Graphs and L-Product with Cycles by Stars Graphs

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Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$, and let $\mathbb{Z}_2 = \{0, 1\}$. A labeling $f : E(G) \rightarrow \mathbb{Z}_2$ of a graph $G$ is said to be edge-friendly if $|\{e_f(0) - e_f(1)\} | \leq 1$. An edge-friendly labeling $f$ induces a partial vertex labeling $f^+ : V(G) \rightarrow \mathbb{Z}_2$ defined by $f^+(x) = 0$ if the number of edges labeled by 0 incident on $x$ is more than the number of edges labeled by 1 incident on $x$. Similarly, $f^+(x) = 1$ if the number of edges labeled by 1 incident on $x$ is more than the number of edges labeled by 0 incident on $x$. $f^+(x)$ is not defined if the number of edges labeled by 1 incident on $x$ is equal to the number of edges labeled by 0 incident on $x$. For $i \in \mathbb{Z}_2$, let $v_f(i) = \text{card}\{v \in V(G) : f^+(v) = i\}$ and $e_f(i) = \text{card}\{e \in E(G) : f(e) = i\}$. The edge-balance index set of the graph $G$, EBI($G$), is defined as $\{v_f(0) - v_f(1) : e_f(0) - e_f(1)\}$ for the edge labeling $f$ is edge-friendly. The edge-balance index sets of Centipede graphs and L-Product with Cycles by Stars graphs are presented in this paper.

The Sequence Maker-Breaker Game

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The sequence maker-breaker game is played on a graded poset by two players, Maker and Breaker, who take turns claiming elements of the poset for themselves. Maker’s goal is to claim a large chain in the poset, claiming these elements in an order which agrees with the chain ordering. Breaker aims to prevent this. More precisely, we will say that Breaker wins at level $k$ if Breaker can prevent Maker from claiming any chain which contains elements at levels $\{0, 1, \ldots, k\}$; we will say that Maker wins if there is no $k$ such that Breaker wins at level $k$ (i.e., if Maker can claim arbitrarily long chains). We show that this form of the game is equivalent to a generalized version of Conway’s famous angel-devil game, and use these ideas to find posets on which Maker wins and to prove results about biased versions of the game.
Multi-Deletion Reconstruction Numbers of Small Graphs

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In 1985 Harary and Plantholt described the concept of a Graph Reconstruction Number: the number of 1-vertex-deleted subgraphs of $G$ required to uniquely identify $G$ up to isomorphism. This concept can be extended in the obvious way to both $k$-vertex-deleted subgraphs and $k$-edge-deleted subgraphs.

In a recent paper we reported the distribution of 1-vertex and 1-edge deleted reconstruction numbers for all graphs on up to 11 vertices. Now we present the results of computation of many $k$-vertex and $k$-edge reconstruction numbers of graphs up to 9 vertices.

On Deciding Whether the Distinguishing Chromatic Number of a Graph is at most Two

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A vertex $k$-coloring of graph $G$ is distinguishing if the only automorphism of $G$ that preserves the coloring is the identity automorphism. The distinguishing chromatic number of graph $G$, denoted $\chi_D(G)$, is the smallest positive integer $k$ such that $G$ admits a proper $k$-coloring that is distinguishing.

Cheng recently showed that when $k \geq 3$, the problem of deciding whether the distinguishing chromatic number of a graph is at most $k$ is NP-hard. We consider the problem when $k = 2$. In regards to the issue of solvability in polynomial time, we show that the problem is at least as hard as graph automorphism but no harder than graph isomorphism.

Putting Dots in Triangles

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Consider a “triangle” of $n(n + 1)/2$ unit squares in a grid. Denote by $N(n)$ the maximum number of dots that can be placed into the cells of the triangle such that each row, each column, and each diagonal parallel to the third side of the triangle contains at most one dot. We prove that $N(n) = \left\lfloor \frac{2n+1}{3} \right\rfloor$ for all positive integers $n$. Here is an (optimal) solution for $n = 7$:

It was proven by Vaderlind, Guy and Larson (2002) and independently by Nivasch and Lev (2005) that $N(n) = \left\lfloor \frac{2n+1}{3} \right\rfloor$. We give a new proof of this result using a linear programming approach.
On Balance Index Sets of Trees of Diameter Four

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Let $G$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$, and let $\mathbb{Z}_2 = \{0, 1\}$. A labeling $f : V(G) \to \mathbb{Z}_2$ induces an edge partial labeling $f^* : E(G) \to \mathbb{Z}_2$ defined by $f^*(uv) = f(u)$ if and only if $f(u) = f(v)$ for each edge $uv \in E(G)$. For $i \in \mathbb{Z}_2$, let $v_f(i) = |\{v \in V(G) : f(v) = i\}|$ and $e_f(i) = |\{e \in E(G) : f^*(e) = i\}|$. We call $f$ is a friendly labeling if $|v_f(0) - v_f(1)| \leq 1$. The balance index set of $G$, denoted BI($G$), is defined as $\{|e_f(0) - e_f(1)| : the vertex labeling $f$ is friendly.$\}$. In this paper, we study the balance index sets of graphs which are trees of diameters four.

Generalized Leech trees

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Let $T$ be a tree on $n$ vertices. Given an labeling $w : E(T) \to \mathbb{Z}^+$ of the edges of $T$, define the weight of a path on $T$ as the sum of the labels on its component edges. We say that $w$ is a Leech labeling of $T$ if the set of path weights is precisely $\{1, 2, \ldots , n/2\}$. A Leech tree is a tree together with such a labeling. In this talk we introduce a generalization of this concept: instead of positive integers, take the labels from some finite Abelian group $(G, +)$. Our analogue of the Leech condition is that the set of path weights must be precisely $G - \{0\}$. We explore this for general groups, with particular emphasis on the case $G = C_2^k$.

Disjoint Cycles with Prescribed Lengths and Independent Edges

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We conjecture that if $k \geq 2$ is an integer and $G$ is a graph order $n$ with minimum degree at least $(n + 2k)/2$, then for any $k$ independent edges $e_1, \ldots , e_k$ in $G$ and for any integer partition $n = n_1 + \cdots + n_k$ with $n_i \geq 4(1 \leq i \leq k)$, $G$ has $k$ disjoint cycles $C_1, \ldots , C_k$ of orders $n_1, \ldots , n_k$, respectively such that $C_i$ passes through $e_i$ for all $1 \leq i \leq k$. We show that this conjecture is true for the case $k = 2$. The minimum degree condition is sharp in general.
Constructions of \((a, k)\)-Strongly Indexable Graphs from \((1, k)\)-Strongly Indexable Graphs

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For any integer \(a, k \geq 1\), a graph \(G\) with vertex set \(V(G)\) and edge set \(E(G)\), \(p = |V(G)|\) and \(q = |E(G)|\), is said to be \((a, k)\)-strongly-indexable (in short \((a, k)\)-SI) if there exists a function pair \((f, f^+)\) which assigns integer labels to the vertices and edges, i.e., \(f : V(G) \rightarrow \{0, 1, \ldots, p - 1\}\) and \(f^+ : E(G) \rightarrow \{a, a + k, a + 2k, \ldots, a + (q - 1)k\}\) are onto, where \(f^+(u, v) = f(u) + f(v)\) for any \((u, v) \in E(G)\).

We determine here classes of graphs that are \((a, k)\)-SI graphs which are derivable from \((1, k)\)-SI graphs.

Nonhomogeneous Nowhere Zero Flows in Line Graphs

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Tutte in 1950s showed that a plane graph \(G\) has a face \(k\)-coloring if and only if \(G\) has a nowhere zero \(A\)-flow, for any abelian group \(A\) with \(|A| > k\). Jaeger et al extended the notion of nowhere zero flows to nowhere zero \((A, b)\)-flows, and then defined the \(A\)-connectedness. Let \(Z_3\) denote the cyclic group of order 3. And also Jaeger et al proposed a conjecture that every 5-edge-connected graph is \(Z_3\)-connected. I will talk about that this conjecture holds for all 5-edge-connected graphs if and only if every 5-edge-connected line graph is \(Z_3\)-connected. As supporting evidences to this conjecture of Jaeger et al, we prove that every 6-edge-connected triangulated line graph is \(Z_3\)-connected. Moreover, by using Ryjacek’s line graph closure, we also prove that every 7-edge-connected triangulated claw-free graph is \(Z_3\)-connected.

Steganosis Using Orthogonal Wavelets With Conditional Probability and Primitive Polynomials

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In this paper we introduce a theory to perform steganosis (transmit messages using images, video, and or audio). We devise an orthogonal reversible wavelet transformation. Depending on the size of the message we embed the message in an image video or audio without altering the original signal, using combinatorics and conditional probability theory. We devise the key using an algorithm similar to the advanced encryption standard, using primitive polynomials and matrix transformations. We present two methods one that the key has already been transmitted and the other that the encrypted key is hidden in the image or video along with the signal. To assure that the integrity of the transmitted signal is not lost we apply lossless compression along with redundancy that includes checksums and forward error correction.
A k-ranking of a graph is a labeling of the vertices with positive integers 1, 2, ..., k so that every path connecting two vertices with the same label contains a vertex of larger label. An optimal ranking is one in which k is minimized. Let G be a graph containing a Hamiltonian path on vertices v_1, v_2, ..., v_n but no Hamiltonian cycle. We use a greedy algorithm to successively label the vertices assigning each vertex with the smallest possible label that preserves the ranking property. We show that when G is a path the greedy algorithm generates an optimal k-ranking. We then investigate two generalizations of rankings. We first consider bounded (k, s)-rankings in which the number of times a label can be used is bounded by a predetermined integer s. We then consider k_t-rankings where any path connecting two vertices with the same label contains t vertices with larger labels. We show for both generalizations that when G is a path, the analogous greedy algorithms generate optimal k-rankings. We then proceed to quantify the minimum number of labels that can be used in these rankings. We define the bounded rank number χ_{r,s}(G) to be the smallest number of labels that can be used in a (k, s)-ranking and show for n ≥ 2, χ_{r,s}(P_n) = \left\lceil \frac{n-(s^2-1)}{2} \right\rceil + i where i = \lceil \log_2(s) \rceil + 1. We define the k_t-rank number, χ_t^r(G) to be the smallest number of labels that can be used in a k_t-ranking. We present a recursive formula that gives the k_t-rank numbers for paths, showing χ_t^r(P_j) = n for all a_{n-1} < j ≤ a_n where \{a_n\} is defined as follows: a_1 = 1 and a_n = \left\lceil \frac{t+1}{t+1}a_{n-1} \right\rceil + 1.

\[ K^-_5\text{-factor in a Graph} \]

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Let G be a graph and let δ(G) denote the minimum degree of G. Let F be a given connected graph. Suppose that |V(G)| is a multiple of |V(F)|. A spanning subgraph of G is called an F-factor if its components are all isomorphic to F.

In 2002, Kawarabayashi conjectured that if G is a graph of order ℓk(ℓ ≥ 3) with δ(G) ≥ \frac{ℓ^2-3ℓ+1}{2}k, then G has a \(K^-_ℓ\)-factor, where \(K^-_ℓ\) is the graph obtained from \(K_ℓ\) by deleting just one edge. In this paper, we prove that this conjecture is true when \(ℓ = 5\).
Rainbow Connectivity of Graphs

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An edge-colored tree $T$ is a rainbow tree if no two edges of $T$ are colored the same. For a nontrivial connected graph $G$ of order $n$ and an integer $k$ with $2 \leq k \leq n$, a $k$-rainbow coloring of $G$ is an edge coloring having the property that for every set $S$ of $k$ vertices of $G$, there is a rainbow tree $T$ containing the vertices of $S$. The rainbow connectivity of a graph is defined in terms of rainbow trees. Some results are presented on rainbow connectivity of a graph.

Second Neighborhood of Triangle-Free Digraphs

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Let $D$ be a simple digraph without digons. For any $v \in V(D)$, let $d^+(v)$ be the number of vertices with out-distance 1 from $v$, and $d^{++}(v)$ be the number of vertices with out-distance 2 from $v$. It was conjectured that for any digraph $D$, there exists a vertex $v$ such that $d^{++}(v) \geq d^+(v)$. Chen et. al. proved that for any digraph $D$, there exists a vertex $v$ such that $d^{++}(v) \geq \gamma d^+(v)$, where $\gamma = 0.657 \cdots$ is the unique real root of $2x^3 + x^2 - 1 = 0$. In this paper, we showed that for any digraph $D$ without directed 3-cycles, there exists a vertex $v$ such that $d^{++}(v) \geq r d^+(v)$, where $r = 0.675 \cdots$ is the positive real root of $x^3 + 3x^2 - x - 1 = 0$.

Characterization of Induced Matching Extendable Graphs with $2n$ Vertices and $3n$ Edges

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A graph $G$ is induced matching extendable or IM-extendable if every induced matching of $G$ is contained in a perfect matching of $G$. In 1998, Yuan proved that a connected IM-extendable graph on $2n$ vertices has at least $3n - 2$ edges, and that the only IM-extendable graph with $2n$ vertices and $3n - 2$ edges is $T \times K_2$, where $T$ is an arbitrary tree on $n$ vertices. In 2005, Zhou and Yuan proved that the only IM-extendable graph with $2n \geq 6$ vertices and $3n - 1$ edges is $T \times K_2 + e$, where $T$ is an arbitrary tree on $n$ vertices and $e$ is any edge connecting two vertices that lie in different copies of $T$ and have distance 3 between them in $T \times K_2$. In this paper, we introduced the definition of $Q$-joint graph and characterized the structure of the connected IM-extendable graphs with $2n \geq 4$ vertices and $3n$ edges.