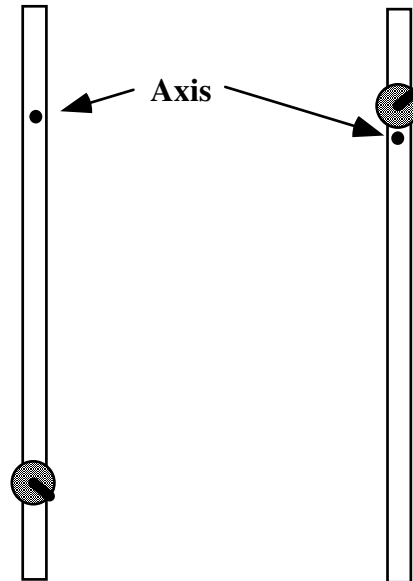


Moment of Inertia (a.k.a. Rotational Inertia) and the Parallel Axis Theorem

311 Review: Consider Newton’s second law. It can be thought of as a way to define mass: mass is the proportionality constant that relates the net force acting on an object and the acceleration of that object. Suppose you hit two different masses with the same net force, which one will move the most, the one with the **larger** mass or the one with the **smaller** mass? _____

Tape some 50 or 100 gram masses to a meter stick, less than 20 cm from the end. Then hold it at one “axis,” 20cm from the end, and rotate it. Now hold it at the other “axis,” 20 cm from the other end. Try to rotate it by applying the force at the same distance from the axis with the same force. Gently swing it back and forth like a sword to get the feel of it. Does it resist being swung (Angular acceleration) more or less now compared to the first case?



Everyone should try this, and everyone should watch while their partners try it. Take care to try to move it with the same force or ‘twist’ (torque) in both cases.

The equivalent of Newton’s second law for rotation written out in words is:

Net Torque = Moment of Inertia * Angular Acceleration.

This can be thought of as a way to define moment of inertia: moment of inertia is the proportionality constant that relates the

_____ acting on an object to the _____ of that object.

If the moment of inertia is the proportionality constant relating torque [N·m] and angular acceleration, what must its units be in terms of kg, meter, and sec?

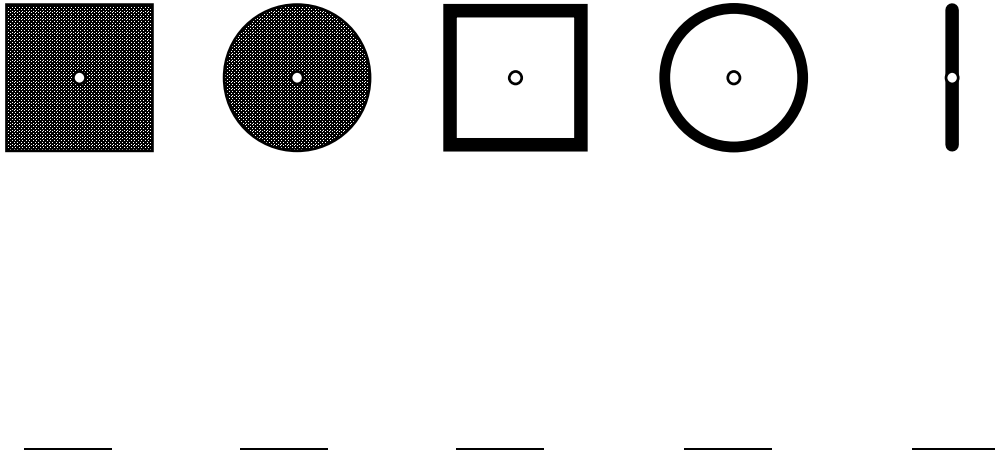
Dimensional Analysis: (You can tell a lot about something just by looking at the units).

In which case was the moment of inertia larger, when the masses were close to the axis or when they were far from the axis of rotation?

If you attached more mass to the meter stick, would you expect the total moment of inertia, I , to increase or decrease? If you doubled the mass, how would I change? (ignoring the mass of the stick)

If you moved the masses **farther from** the **axis** of rotation, would you expect its moment of inertia, I , to **increase** or **decrease**? If you doubled the distance, how would I change?

Brain-Buster: Diagrams of 5 objects (a solid square, a solid disk, a hollow square, a hoop, and a thin rod) are shown below. The center is the axis of rotation (out of the page) around which they rotate. The objects all have the same total mass, and the vertical dimension is the same for all of them.



Without looking at any formulas from the table in the book just yet, rank the objects from smallest [= 1] to largest [= 5] moment of inertia. Explain how you made your ranking.

Given a cylinder and a sphere, both of the same radius and mass, which one has the smaller moment of inertia?

Something to be careful of: If the moment of inertia I depends in general upon the axis of rotation through any given object, is the moment of inertia a vector or a scalar?

The text has a table of the moments of inertia of various simple objects provided the objects are rotated about their centers of mass. We can find the moment of inertia about points other than the center of mass of an object using something called the Parallel Axis Theorem. We will revisit this later, but it is simple to state and start using now. The Parallel Axis theorem states;

$$I = I_{\text{com}} + Mh^2$$

where h is the distance from the center of mass to the axis of rotation, and I_{com} is the known center of mass (from the tables) about an axis parallel to the axis of rotation.

Q1) Use the parallel axis theorem and a formula from the textbook to find a formula for the moment of inertia I about one end for a uniform rod of length L and mass m .

Q2) Consider holding a meter stick horizontally. It is pivoted at one end and released from rest. What will be the angular speed of the meter stick when it is momentarily vertical?

