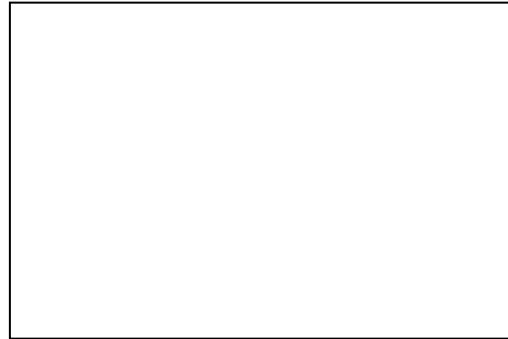


Calculating Moments of Inertia, Part I

What is the simplest object that you can think of, something that all other more complicated objects can be built out of, simpler than a sphere, rod or any of the basic shapes we've briefly looked at so far? _____

Draw a picture of this situation using the notation for drawing three dimensions on a piece of paper:



Review: The magnitude of the torque τ in terms of F_T (the tangential force) and r (technically called the 'lever arm') is: _____

1) Starting with the above relationship, and knowing that $F_T = ma_T$, figure out what the moment of inertia I must be for this simplest of all cases.

Answer:

We can build any more complicated objects out of point masses. Torque is a vector, so a number of small torques all in the same direction will add up to one large torque in that direction. If the object is rigid, we can write

$$\tau_{\text{net}} = \sum (m_i r_i^2 \alpha) = \sum (m_i r_i^2) \alpha \quad \text{and then it follows that} \quad I_{\text{total}} = \sum m_i r_i^2$$

Q: Why must the object be rigid for this to work? _____

Now take this expression for I_{total} and write it as an integral expression (hint: which piece becomes infinitesimal if we sum over a whole bunch of very small pieces):

$$I_{\text{total}} = \underline{\hspace{10em}}$$

We will work out this integral for objects in subsequent workshops. However, there are a few cases which can be easily worked out without having to use the calculus;

2) Consider a hollow thin-walled cylinder of radius R and mass M rotated about the cylinder axis. Draw a sketch of this situation. Start with either the sum or integral expressions above and calculate what the moment of inertia must be. There are two simple logical steps here, and be sure to show clearly what they are and what order you're doing them in.

Answer: