Team Physics 312 Name: ____________________________

Activity 3A

Date: ___________ Table/Team: _________________________

Calculating Moments of Inertia, Part II
(1-Dimensional Integrations)

1) Assume we have a long thin rod of uniform linear density $\lambda = m/L$. This is the conventional symbol for a linear density, and would be expressed in, for example, [Kg/m]. Evaluate the integral

$$I_{com} = \int r^2 \, dm$$

for a long thin rod as shown in your table of moments of inertia. First relate dm to dx and then integrate along the length of the rod from $x = -L/2$ to $x = L/2$. Does the result agree with that given in the tables?

2) Now that we've derived the formula in the I table, change the limits of integration to the interval $x = 0$ to $x = L$. This would be the case of a rod rotating about one of the ends. Does the result agree with the parallel-axis theorem? (This shows you have to be careful to have the coordinate zero where the axis of rotation is when calculating I).

3) Again, change the limits of integration to the limits $x = R - L/2$ to $x = R + L/2$. What physical situation does this describe? Does this result also agree with the parallel-axis theorem? What does the result look like in the extreme case of $R >> L$?

4) **Brain-Buster:** Imagine that the long thin rod is now at an angle θ with respect to the z-axis of rotation, not necessarily 90º as before. What will be I_{com} in this case? (Hint: you can compute this directly, just be careful that r is the perpendicular distance from dm to the axis of rotation, or you could use the conceptual trick of 'squishing it flat' along the axis of rotation, which doesn't change I about that axis).

5) **Group Project:** Now we have a long thin rod of non-uniform linear density given by the function

$$\lambda = 2 + (x - (L/2))^2/(3L^2) \quad [\text{ kg/m}]$$

where x = 0 is one end of the rod of length L. (This could be a rod made of an alloy that has a variable density, or more commonly we may have a rod of uniform density but varying thickness, and it may be convenient to represent this mathematically by a 1-d thin rod of varying density).

a) First, write the total mass M of the rod, in terms of L.

b) Calculate the moment of inertia of the rod about its center $x = L/2$ as in question #1. Write this as some fraction times ML^2. (Hint: use the result from (a) to eliminate one of the L terms. This is essentially the same as what was done in the previous problems eliminating λ from the final answer, because the fraction in (a) has units of kg/m).

A nice check of your answer here is that it should be roughly similar to that of a uniform long thin rod, because the equation for $\lambda(x)$ just makes it a little denser at the ends.