

### Calculating Moments of Inertia, Part III (2- and 3-Dimensional Integrations)

Recall in part II we had a long thin rod that rotated about an axis through its center and perpendicular to its length. We calculated the moment of inertia of this rod by using the formula

$$I = \int dI = \int r^2 dm \quad \text{where } r \text{ is the distance of the mass } dm \text{ from the axis.}$$

We follow a similar approach in 2 and 3 dimensions, but typically we build the integral out of results from lower dimensions to cut down on the difficulty of evaluating the integrals. Today's worksheet will go through several examples of this technique.

In general, the way you should approach these problems is to:

- Draw a clear diagram showing your object and the axis of rotation.
- Pick a "slice" (or piece)  $dm$  at constant  $r$ , which is not at the middle or the end of your object, and show  $r$  on your diagram. Alternatively, you can also sometimes write a  $dI$  directly in terms of simpler ones you already know from a table.
- Get a formula for  $dm$  or  $dI$  in terms of a variable you can integrate, as we did on part II.

Note that the conventional symbols for density in different dimensions are

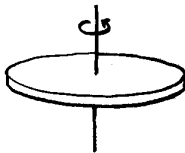
$$\lambda = M/L \text{ for 1-dimensional objects.} \quad \rightarrow \quad dm = \lambda dx$$

$$\sigma = M/A \text{ for 2-dimensional objects.} \quad \rightarrow \quad dm = \sigma dA$$

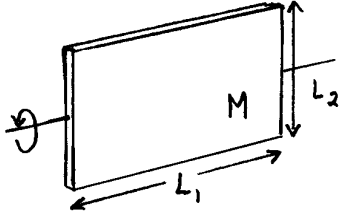
$$\rho = M/V \text{ for 3-dimensional objects.} \quad \rightarrow \quad dm = \rho dV$$

- Substitute for  $dm$  or  $dI$  in your integral formula.
- Do the integral.

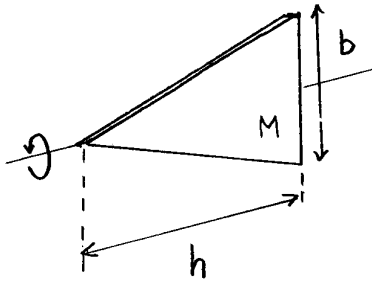
1) First try using this method to find  $I$  for a flat, circular disk (or cylinder, same result) with mass  $M$  and radius  $R$ , rotating about an axis through the center as shown below. Build upon a result already known from part II, the thin-walled cylinder.



2a) Determine the moment of inertia of a thin rectangular slab as shown below. There are a couple of ways to make the slices, so if you finish early, try making the cuts in the other direction to make sure you get the same answer.



2b) Determine the moment of inertia of a thin triangular slab as shown below. This is only slightly trickier, and is leading up to the final problem of part III. There are still a couple of ways to make the slices, but the answer should be the same either way.



2c) **Brain-Buster:** If you finished the previous problems early, try finding the moment of inertia of a disk rotating about an axis through the disk as shown below. You'll probably need integral tables to solve the actual integral, or ask your prof to write down the integral needed after you've set up the problem.



Team Physics 312  
Workshop 4A  
Date: \_\_\_\_\_

Names: \_\_\_\_\_  
Table/Team: \_\_\_\_\_

Group project for this workshop: Find the moment of inertia for a solid circular cone of mass  $M$ , length  $L$  and radius  $R$  at the base. The cone is to be rotated about its axis of symmetry. (A simple 3-d shape, a simple result, but one that is not in your text tables so you can't peek ahead and know the answer). Write one solution per group neatly on this separate piece of paper to hand in at the end of the class.

