

**Superposition, Phasors and Standing Waves****1. Equal amplitudes, same direction, same frequency and wavelength**

Use the waves  $y_1 = y_m \sin(kx - \omega t)$  and  $y_2 = y_m \sin(kx - \omega t + \phi_0)$

If they are equal amplitude, then the sum can be obtained from the trig identity:

$$\sin \alpha \pm \sin \beta = 2 \sin \left[ \frac{\alpha \pm \beta}{2} \right] \cos \left[ \frac{\alpha \mp \beta}{2} \right]$$

**Q1:** Add  $y_1$  and  $y_2$  using this trig relation.

**Q2:** Could we still use this identity somehow if they are not of equal magnitude?

Note that in your solution to Q1, depending on the phase constant, the interference can be

Completely (or totally) constructive, amplitude =  $2y_m$  when  $\phi_0 = 0, 2\pi, 4\pi, \dots$

Completely (or totally) destructive, amplitude = 0 when  $\phi_0 = \pi, 3\pi, 5\pi, \dots$

**Q3:** Go to the applet same Physlet we had yesterday;

[http://people.rit.edu/vwlsp/312\\_s03/Physlets/pulseSuperposn.html](http://people.rit.edu/vwlsp/312_s03/Physlets/pulseSuperposn.html)

and verify that changing the phase by any amount still gives a traveling wave that has the same velocity (magnitude and direction), the same frequency, and the same wavelength.

Problems:

**Q4.** Write an expression for the superposition of

$$y_1 = 3.0 \text{ mm} \sin(kx - \omega t) \quad \text{and} \quad y_2 = 3.0 \text{ mm} \sin\left(kx - \omega t + \frac{\pi}{3}\right)$$

**Q5.** The superposition of

$$y_1 = 3.0 \text{ mm} \sin(kx - \omega t) \quad \text{and} \quad y_2 = 3.0 \text{ mm} \sin(kx - \omega t + \phi_0)$$

has amplitude of 4.0 mm. What is the phase constant  $\phi_0$ ?

Verify that your answers to Q4 and Q5 match the correct waves using the applet linked above.

## **2. Standing Waves (Equal amplitudes, opposite directions, same frequency and wavelength)**

These waves can be represented by

$$y_1 = y_m \sin(kx - \omega t) \quad \text{and} \quad y_2 = y_m \sin(kx + \omega t)$$

**Q6.** Add these using the same trig relation to show that you get:  $y_1 + y_2 = 2y_m \sin kx \cos \omega t$

**Q7.** How do we know that this is no longer a traveling wave?

**Q8.** Explain how we can find the locations of nodes [where the amplitude is always zero] along the x-axis.

**Q9.** To find antinodes [where the amplitude is a maximum], we must set  $kx$  equal to \_\_\_\_\_

**Old exam questions:**

**Q10.** i) On the axes below, sketch graphs of the sum of

$$y_1 = y_m \sin (kx - \omega t) \text{ and } y_2 = y_m \sin (kx + \omega t)$$

at times  $t = 0$ ,  $t = T/2$ , and  $t = 3T/4$ . Label the three graphs to identify which one is which.



ii) If the wave is fixed at  $x=0$  and  $x=L$ , what are the possible values of the wavenumber  $k$ ?

**Q11.** Find the result of adding these three waves in linear superposition;

$$y_1 = (1.0 \text{ cm}) \sin (kx + \omega t)$$

$$y_2 = (2.0 \text{ cm}) \sin [(kx + \omega t) + 2\pi/3]$$

$$y_3 = (3.0 \text{ cm}) \sin [(kx + \omega t) + 4\pi/3]$$