Math/Stat Seminar

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An Introduction to MAPLE

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Definition: The ***factorial*** of n, denoted $n!$, is defined for nonnegative integers n as

$$n! = \begin{cases} 1\cdot 2\cdot 3\cdot\cdots\cdot(n-1)n, & \text{if } n>0; \\ 1, & \text{if } n=0. \end{cases}$$

Problem #1: Determine the number of zeros at the end of the following numbers.

(*a*) 10!
(*b*) 100!
(*c*) 1000!

Using MAPLE:

```
> 1*2*3*4*5*6*7*8*9*10;
```

OR

```
> 10!;
```

Initial output:

3628800

Bonus: Find a formula for the number of zeros at the end of $n!$.

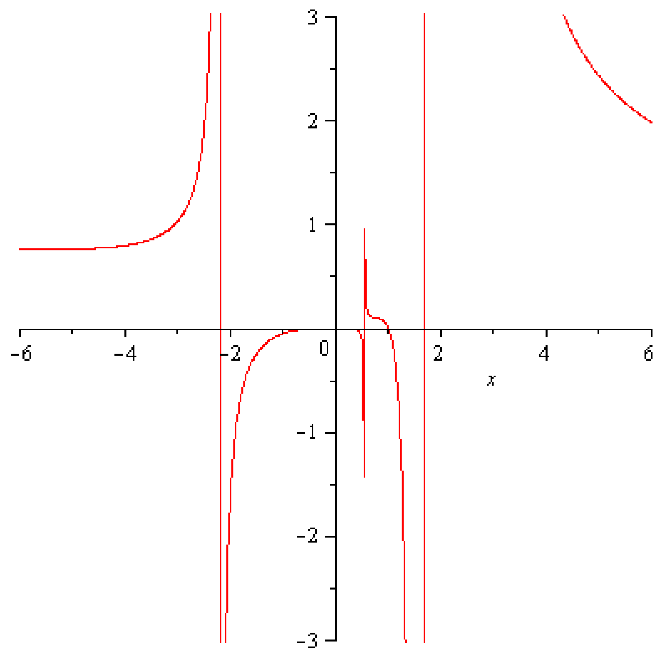
Problem #2 (*courtesy of Professor Bautista*): Find the x and y coordinate of the local minimum in the function

$$f(x) = \frac{x^5 - x^4}{(x^3 - 4x + 2)(x - \frac{19}{10})^2}.$$

Using MAPLE:

```
> with(plots);
> f := x -> (x^5-x^4)/(x^3-4*x+2)/(x-19/10)^2;
  a := -6:
  b :=  6:
  c := -3:
  d :=  3:
  plot(f(x), x=a..b, view=[a..b,c..d], numpoints=100);
```

Initial output:



Problem #3: Determine the shape of the three dimensional surface defined implicitly as

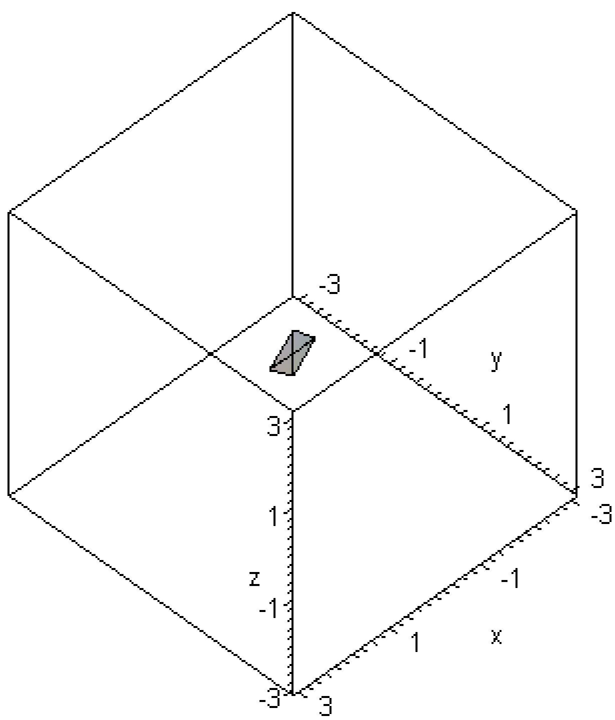
$$\left(x^2+\tfrac{9}{4}y^2+z^2-1\right)^3-x^2z^3-\tfrac{9}{80}y^2z^3=0.$$

What are the dimensions of the most efficient viewing box for this surface?

Using MAPLE:

```
> implicitplot3d((x^2+9/4*y^2+z^2-1)^3 - x^2*z^3 -
  9/80*y^2*z^3 = 0, x=-3..3, y=-3..3, z=-3..3,
  numpoints=20, labels=[x,y,z], axes=box,
  scaling=CONSTRAINED);
```

Initial output:



The Collatz Conjecture

Consider the function

$$H(n) = \begin{cases} \frac{1}{2}n, & \text{if } n \text{ is even;} \\ 3n+1, & \text{if } n \text{ is odd,} \end{cases}$$

defined on the set of positive integers.

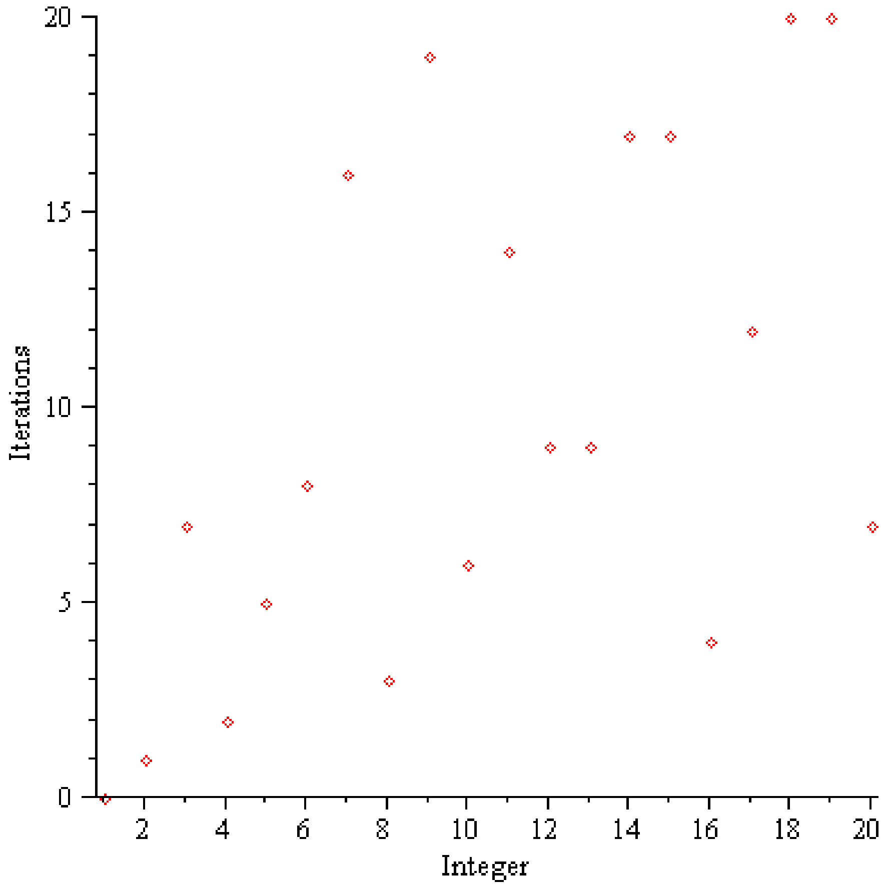
The Collatz Conjecture: All positive integers iterated through H must terminate at 1.

Let $H^*(n)$ be the minimum number of iterations m that are required such that $H^{(m)}(n) = 1$ where $H^{(m)} = \underbrace{H \circ H \circ H \circ \cdots \circ H}_{m \text{ times}}$.

Problem #4: Describe the behavior of $H^*(n)$ by looking at the scatterplot for as many values of n that you can use.

```
> Collatz := proc( m,n::nonnegint )
    local k,x,t;
    global V;

    for k from m to n do
        x := k;
        V[k]:=0;
        while x > 1 do
            if x mod 2 = 0 then x:=x/2 else x:=3*x+1 fi:
            V[k]:=V[k]+1:
        od:
    od:

  end proc:

  m:=1: n:=20:

  Collatz(m,n);

   plot([seq( [k,V[k]], k=m..n )], style = POINT, labels =
     ["Integer", "Iterations"], labeldirections =
     [HORIZONTAL, VERTICAL]);
```

Initial output:



20
15
10
5
0
Iterations
2
4
6
8
10
12
14
16
18
20
Integer