Probability and Statistics Problems - Solutions

1. You have some trick coins that land heads 60% of the time and tails 40%. Use the binomal expansion to calculate the probabilities of HH, HT, and TT. If you flip 2 coins 175 times, what are the numbers of each?

	HH	HT	TT
probability	H^2 $(0.6)^2$ 0.36	2HT 2(0.6)(0.4) 0.48	T^2 $(0.4)^2$ 0.16
175 times	0.36 x 175 = 63	0.48 x 175 = 84	0.16 x 175 = 128
230 times	0.36 x 230 = 82.8	0.48 x 230 = 110.4	0.16 x 230 = 36.8

2. A trick penny lands heads 35% of the time and a trick nickel lands heads 55% of the time. Use the binomial expansion to find the probabilities of the various combinations of heads and tails. What would be the numbers if you flipped them 275 times? 465 times?

Trick Penny: H = 35% T = 65% Trick Nickel: h = 55% t = 45% (H+T) x (h+t):

	P H N H	P H N T	P T N H	P T N H
probability	(0.35)(0.55) 0.193	(0.35)(0.45) 0.158	(0.65)(0.55) 0.358	(0.65)(0.45) 0.29
275 times	0.193 x 275 = 53	0.158 x 275 = 43.5	0.358 x 275 = 98.5	0.29 x 275 = 79.8
465 times	0.193 x 465 = 89.8	0.158 x 465 = 73.5	0.358 x 465 = 166.5	0.29 x 465 = 134.9

The remaining questions have to do with the frequencies of marbles in the following Jars

Jar 1	**Jar 2**	**Jar 3**	**Jar 4**
80 blue = 0.8 20 red = 0.8	70 green = 0.7 30 yellow = 0.3	50 black = 0.5 50white = 0.5	50 orange = 0.5 50 brown = 0.5

3. Use the binomial expansion to calculate the probabilities and combinations by drawing from the jars as indicated:

Jars 3 + 4 (Bl + W) (O + Br)	combinations	BlO	BlBr	WO	WBr
	probabilities	0.5 x 0.5 = 0.25	0.5 x 0.5 = 0.25	0.5 x 0.5 = 0.25	0.5 x 0.5 = 0.25

Jars 1 + 3 (B + R) (Bl + W)	combinations	BBl	BW	RBl	RW
	probabilities	0.8 x 0.5 = 0.4	0.8 x 0.5 = 0.4	0.2 x 0.5 = 0.1	0.2 x 0.5 = 0.1

	combinations	probabilities
Jars 1 + 2 + 3 (B+R)(G+Y)(Bl+W)	BGBl	0.8 x 0.7 x 0.5 = 0.28
	BGW	0.8 x 0.7 x 0.5 = 0.28
	BYBl	0.8 x 0.3 x 0.5 = 0.12
	BYW	0.8 x 0.3 x 0.5 = 0.12
	RGBl	0.2 x 0.7 x 0.5 = 0.07
	RGW	0.2 x 0.7 x 0.5 = 0.07
	RYBl	0.2 x 0.3 x 0.5 = 0.03
	BYW	0.2 x 0.3 x 0.5 = 0.03

4. Calculate the probabilities for each event below:

pulling 10 blue and 10 red from jar 1

$$P=\frac{n!}{X!(n-X)!}\ p^{X}q^{(n-X)}=\frac{20!}{10!\times 10!}\ 0.8^{10}\times 0.2^{10}=\frac{2.43\times 10^{18}}{1.32\times 10^{13}}\ (1.07\times 10^{-1})\times(1.07\times 10^{-1})=0.0018$$

pulling 25 green and 18 yellow from jar 2

$$P=\frac{n!}{X!(n-X)!}\ p^{X}q^{(n-X)}=\frac{43!}{25!\times 18!}\ 0.7^{25}\times 0.3^{18}=\frac{6.04\times 10^{52}}{1.02\times 10^{41}}\ (1.3\times 10^{-4})\times(3.87\times 10^{-10})=0.03$$

pulling 6 black and 15 white from jar 3

$$P=\frac{n!}{X!(n-X)!}\ p^{X}q^{(n-X)}=\frac{21!}{6!\times 15!}\ 0.5^{6}\times 0.5^{15}=\frac{5.1\times 10^{19}}{9.36\times 10^{14}}\ (0.016\times 10^{-1})\times(3.05\times 10^{-5})=0.027$$

pulling 10 orange and 10 brown from jar 4

$$P=\frac{n!}{X!(n-X)!}\ p^{X}q^{(n-X)}=\frac{20!}{10!\times 10!}\ 0.5^{10}\times 0.5^{10}=\frac{2.43\times 10^{18}}{1.3\times 10^{13}}\ (9.7\times 10^{-4})\times(9.7\times 10^{-4})=0.176$$

5. You draw from jars 2 and 4 200 times and get the results below. Use X^2 to determine whether or not this result is expected.

Results	Observed	Expected	Obs - Exp	(Obs - Exp)^2	÷ Exp
green + orange Ex = 0.7 x 0.5 x 200	65	70	-5	25	0.357
green + brown Ex = 0.7 x 0.5 x 200	75	70	5	25	0.357
yellow + orange Ex = 0.3 x 0.5 x 200	28	30	-2	4	0.133
yellow + brown Ex = 0.3 x 0.5 x 200	32	30	2	4	0.133
Totals	200	200		X^2= 0.98	

Null Hypothesis = There is no difference between observed and expected results

Degrees of Freedom = 3

Probability Level = 0.05

Critical X2 = 7.815

[X] Accept H_o

[ ] Reject H_o

6. You draw from jars 1 and 2 450 times and get the results below. Use X^2 to determine whether or not this result is expected.

Results	Observed	Expected	Obs - Exp	(Obs - Exp)^2	÷ Exp
blue +green Ex = 0.8 x 0.7 x 450	230	252	-22	484	1.92
red + yellow Ex = 0.2 x 0.3 x 450	38	27	11	121	4.48
blue + yellow Ex = 0.8 x 0.3 x 450	115	108	7	49	0.45
red + green Ex = 0.2 x 0.7 x 450	67	63	4	16	0.25
Totals	450	450		X^2= 7.1	

Null Hypothesis = There is no difference between observed and expected results

Degrees of Freedom = 3

Probability Level = 0.05

Critical X2 = 7.815

[X] Accept H_o

[ ] Reject H_o

7. You draw from jars 1 and 3 600 times and get the results below. Use X^2 to determine whether or not this result is expected.

Results	**Observed**	**Expected**	**Obs - Exp**	**(Obs - Exp)^2**	**÷ Exp**
blue + black Ex = 0.8 x 0.5 x 600	230	240	-10	100	0.42
red + black Ex = 0.2 x 0.5 x 600	70	60	10	100	1.67
blue + white Ex = 0.8 x 0.5 x 600	250	240	10	100	0.42
red + white Ex = 0.2 x 0.5 x 600	50	60	-10	100	1.67
Totals	600	600		X^2= 4.18	

Null Hypothesis = There is no difference between observed and expected results

Degrees of Freedom = 3

Probability Level = 0.05

Critical X2 = 7.815

[X] Accept H_o

[ ] Reject H_o