Due-care standards in a market setting with legal error

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Abstract

We consider a monopoly market setup with legal error, where social welfare is negatively related to a firm’s expected liability costs. In this context, we investigate the optimal location of the due-care standard vis-à-vis any given desired care level. It is found that when both the due-care standard and the penalty multiplier are choice variables, setting the due-care equal to the desired care is unlikely to be optimal and conditions under which it should be made relatively lenient or stringent are obtained. Exogenous restrictions on the penalty multiplier may restrict the extent to which the due-care can be manipulated to improve welfare.
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1. Introduction

In many important activities such as medical care, construction services and production in general there is often a scope for providers to undertake actions that reduce the likelihood of product failure and consequent loss to consumers and/or third parties. These actions are frequently of such a nature that outsiders cannot observe them perfectly. For example, once a building has been constructed, it is almost impossible for buyers, renters or government agencies to verify whether the contractor complied with all construction codes, such as

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those specifying composition of concrete mixtures, size and depth of footing, internal wiring methods and so on. In case of construction, use of monitoring when construction is under way is possible, but that is also imperfect and in other cases such as medical care and production of mass consumption products, effective monitoring is not feasible. As a result, exposure to liability is often used to motivate safety effort. However, even after an accident the available evidence is often imperfect and there is uncertainty about whether there was compliance with regulatory or legal standards. For example, in *McGinnis v. Jim Walter Homes Inc.* a fire in the plaintiff’s house killed his daughter, who then sued the contractor for the loss over defective electrical wiring. While the expert for the plaintiff testified that fire originated in the attic and was due to faulty electrical wiring, in the state inspector’s opinion, the fire originated in a room where children were playing with matches and that was the most probable cause. Following the destruction caused by the fire, there was no clear-cut evidence available to support either theory. Since courts often have to take decisions based on such imperfect evidence, they can make mistakes—they may either convict innocent parties (called type I error in this paper) or they may exonerate guilty parties (called type-II error).

In this paper we study implications of such errors for the due-care standard and the penalty multiplier when both can be jointly determined. In particular, we focus on the relationship between the due-care standard and the optimal desired care level and ask whether the welfare can be improved by making the due-care standard more lenient or stringent relative to a given desired care level and adjusting the penalty multiplier so that the firm exercises the desired care level. We find the answer to be in the affirmative when there is a scope of legal error and welfare is related to the firm’s expected liability cost.\(^1\) For any given exercised care level, we see that if the rate at which *marginal* likelihood of being found liable increases as the due-care standard increases is not very large then the more lenient the due-care standard is relative to the desired care level, the smaller is the expected liability cost.

*Craswell and Calfee (1984, 1986)* and *Boyer, Lewis, and Liu (2000)* are two papers most closely related to our work.\(^2\) Craswell and Calfee show that when there is uncertainty about standards, there can be either undercompliance or overcompliance and that when this is the case, the mean of the standard can be manipulated to improve compliance behavior. However, they do not study joint determination of the mean standard and the penalty multiplier. Since sanctions are not socially costly in their model – welfare is independent of the magnitude of the expected penalty – different mean standard and penalty multiplier combinations that induce any given care level are equivalent. In fact, the same welfare as under the negligence rule can be achieved by adopting the strict liability rule and setting the penalty equal to the harm. In our paper, we consider a market setting which makes sanctions socially costly. For any given care level, an increase in the expected penalty increases the price of the product, which reduces the welfare. We suggest that by jointly manipulating

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1. We assume that the firm’s likelihood of being found liable depends on the difference between the due-care standard and the exercised care level.

2. While in the setting considered here there is no uncertainty about the due-care standard and the party’s exercised care level is subject to misinterpretation, our results hold in the converse situation considered by *Craswell and Calfee (1986)*, where a firm’s exercised care level is perfectly observed but the location of the due-care standard is uncertain and distributed around some mean. In this alternative setting our results can be interpreted as applying to the *mean* of the due-care standard.
the due-care standard and the penalty multiplier, the same care level may be induced with a lower expected penalty cost to the firm. In Boyer et al. (2000), sanctions are socially costly as they can increase the cost of law enforcement. The authors show that when this is the case, it may be desirable to lower the standard and induce a lower care than that which equates marginal cost and marginal benefit of care. The standard is therefore made lenient to induce a lower care, but whether making it more lenient or stringent for any given desired care level (and adjusting the penalty multiplier accordingly) can improve welfare is not explored.

Shavell (1984), Kolstad et al. (1992), and Schmitz (2000) also solve for the optimal regulatory standard in the context of joint use of ex ante regulation and ex post liability. They find that regulatory standards should be more lenient when regulation and liability are used jointly than when the former is used alone. However, the reason for having lenient standards in these papers is to decrease the care exercised by some parties, and not to influence the expected penalty borne by them. As in Craswell and Calfee (1986), the expected penalty borne by the party exercising care is not relevant in these papers. On the other hand, we focus on joint manipulation of the due-care standard and the penalty multiplier to reduce expected penalty corresponding to any desired care level, which may be important in market settings.

The plan of the paper is as follows. The basic set-up is described in Section 2. Section 3 solves for the optimal location of the due-care standard vis-à-vis the desired care in the benchmark case when there is no legal error. Section 4 considers the same problem in the presence of legal error. Section 5 concludes. All proofs are in Appendix A.

2. The model

Let us consider a monopoly setting, where a firm supplies some useful product/service such as construction or medical care. While the product is useful, it is possible that it may fail, which can harm the consumers of the service and/or third parties. The monopolist can exert care $x \in [0, X]$ that reduces the likelihood of product failure and the extent of harm. For any exercised care-level $x$, let $c(x)$ be the monopolist’s marginal cost of production, and $p(x)$ denote the probability of product failure. If the product fails it results in harm $H$. To simplify the analysis, we assume that the level of harm $H$ does not depend on $x$. Unless an accident

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3 In Shavell (1984) and Schmitz (2000) parties are heterogenous and therefore optimal care varies across them.

4 Following are some other papers that have studied other ways of dealing with inefficiencies resulting from legal error: Png (1986) analyzes the use of damages and subsidies to induce efficient care and activity level; Polinsky and Shavell (1989) study how penalty and incentive to sue can be manipulated to induce compliance with the law; Kaplow and Shavell (1994) make accuracy of legal decisions endogenous, study the optimal degree of accuracy and implications for optimal probability of enforcement and sanctions. These papers assume care to be a binary choice – parties either take care or they do not take care – and therefore the question of whether and when due-care standards should be more stringent or lenient than the actual care level we wish to induce, cannot be addressed in these models.

5 Even if expected level of harm or damage depends on $x$, if we restrict the penalty to be a simple linear function of the actual harm (i.e., we restrict it to be of the form $bH$ where $b$ is the penalty multiplier), then all our results go through (see Bhole, 2005). Things may change if more complicated schemes, as in the standard moral hazard literature, are allowed. Such schemes may, however, be difficult to implement in reality.
takes place nothing is known about the care level exercised. Even following an accident, exercised care level \( x \) cannot be ascertained perfectly. Based on imperfect evidence, the court decides if there is sufficient evidence to rule that the firm did not comply with the standard, and if so, the firm is found liable, otherwise it is exonerated. Since the court’s decision is based on imperfect evidence, it can make mistakes—it may either convict the innocent (type I error) or it may exonerate the guilty (type II error). Let us assume that the likelihood of the firm being found liable is zero unless the due-care standard, \( s \), is greater than or equal to some level \( \hat{s} > 0 \). For example, if \( s = 0 \), then it is impossible for the firm to violate the standard and as a result it will never be found liable. Similarly, for a very low due-care standard, conviction may be impossible because the court may not consider it worthwhile to spend time and resources to hear a case in which it believes conviction to be extremely unlikely. For any due-care standard \( s \geq \hat{s} \), let \( \xi(s - x) \) denote the probability that the firm is found liable following an accident. If \( x < s \), then \( 1 - \xi(s - x) \) denotes the probability of type II error and if \( x > s \), then \( \xi(s - x) \) denotes the probability of type I error. Once the firm is found liable, it has to pay damages equal to \( b \) times the realized harm (i.e. \( bH \)), where \( b > 0 \) is the penalty multiplier. It is assumed that the regulatory agency that sets the due-care standard can also choose the penalty multiplier. We make the following assumptions about these functions:

\[
\begin{align*}
    c'(x) > 0 & \quad \text{and} \quad c''(x) > 0, \quad \text{for all } x \in [0, X] \quad \text{(A.1)}
    \\
    p'(x) < 0 & \quad \text{and} \quad p''(x) > 0, \quad \text{for all } x \in [0, X] \quad \text{(A.2)}
    \\
    \xi'(s - x) > 0, & \quad \text{for all } s \geq \hat{s} \text{ and } x. \quad \text{(A.3)}
\end{align*}
\]

Assumption (A.1) says that the marginal cost of production increases at an increasing rate with the care exercised. (A.2) says that as the care level increases, the likelihood of product failure decreases at a decreasing rate. Finally, (A.3) says that the likelihood that the firm is found liable increases with the egregiousness of the violation. Hence, the probabilities of type I and type II errors depend on the firm’s position relative to the due-care standard. Additional properties of function \( \xi(\cdot) \) and their implications for the optimal position of the due-care standard vis-à-vis the desired care level are considered in Section 4.

Suppose there are two types of consumers who differ in their willingness-to-pay for the firm’s product. Refer to those willing-to-pay \( \theta_h \) as the high-valuation (or high-type) consumers and those willing-to-pay \( \theta_l \) (<\( \theta_h \)) as the low-valuation (or low-type) consumers. Out of a total of \( N \) consumers, fraction \( \alpha \) are of type \( \theta_h \) and fraction \( (1 - \alpha) \) are of type \( \theta_l \). While \( \alpha \) is common knowledge, the firm cannot distinguish between the two types of consumers. For simplicity, we also assume that each consumer consumes at most one unit. This assumption implies that the firm has to charge a uniform price to all consumers. It

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6 This assumption is not crucial for our results. What is required is that whenever an investigation is made, the likelihood that it results in a mistake is positive. This is more likely to happen when investigation is made following an accident, say after a building has collapsed.

7 As observed by Craswell (1996), if legal error depends on the egregiousness of violation then there is no general reason for multiplying damages by a factor greater than one. For this reason we do not restrict \( b \) to be greater than 1.
can either charge $\theta_h$ and serve only the high-valuation consumers or charge $\theta_l$ and serve all consumers.

If the harm due to product failure is inflicted on third parties (as is true in case of many environmental accidents) then consumers’ willingness-to-pay can be assumed to be independent of their expectations about the care level. If, however, the harm is inflicted on consumers then their willingness-to-pay will depend on the expected care level. This expectation can be based on past experiences with the firm (their own or those of others) and on information about $s$, $b$, $c(\cdot)$ and $\xi(\cdot)$. Let us assume that at the time of purchase, consumers do not have information about these variables/functions and therefore $\theta_h$ and $\theta_l$ depend at most on past experiences with the firm and are independent of the regulatory agency’s choice of $s$ and $b$.

The foregoing assumptions imply that the gross benefit to the society from any unit produced is $\theta_l$ (respectively, $\theta_h$) when a low-valuation (respectively, high-valuation) consumer consumes the unit and the expected full cost to the society of any unit is $c(x) + p(x)H$, where $c(x)$ is the cost of production and $p(x)H$ is the expected harm. Let us define

$$x^* = \arg\min_x c(x) + p(x)H,$$

i.e. $x^*$ is the care-level that minimizes the expected social cost of producing any unit.

Throughout the paper it will be assumed that

$$\theta_l > c(x^*) + p(x^*)H. \tag{A.4}$$

This assumption says that even in case of a low-valuation consumer, social benefit from production exceeds expected social cost for some care level. Our objective is to study the welfare-maximizing choice of penalty multiplier $b$, due-care standard $s$, and induced care $x$ in the above described setting. Welfare is defined as the sum of consumer surplus and producer surplus. In particular, we are interested in the optimal location of the due-care standard vis-à-vis the desired care level.

3. Benchmark case: no legal error

If courts can perfectly determine the firm’s exercised care level then it is found liable (following an accident) whenever its care level is below the standard and is exonerated otherwise. Defining by $x_b$ the care level that minimizes $c(x) + p(x)bH$, we get that the firm will exercise care $x_b$ whenever $c(x_b) + p(x)_bH < c(s)$ and $s$ otherwise. That is, the firm will violate the standard if its expected cost with a lower care, despite having to pay damages, is smaller than the cost of complying with the standard; otherwise it will comply with the standard. For any $(s, b)$, denote the firm’s choice of care-level by $x_I(s, b)$, and refer to it as the induced care. Similarly, for any $(s, b)$ and induced care $x_I$, let $\tilde{C}(x_I; s, b)$ denote

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8 See Shavell (2004) for a brief account of why customers’ knowledge of risk may be imperfect. It is shown in Bhole (2005) that our qualitative results go through even if we relax this assumption, as long as some fraction of the penalty imposed on the firm is not received by the consumer—for example, lawyers get a fraction of the award when cases are fought on a contingency basis.
the full cost (cost of production + expected liability cost) per unit produced. Therefore, \( \tilde{C}(x_I; s, b) = c(x_I) + p(x_I)bH \) when \( x_I < s \) and \( \tilde{C}(x_I; s, b) = c(x) \) for \( x_I \geq s \). We can use function \( \tilde{C}(\cdot) \) and welfare expression in (1) below, to determine the firm’s pricing behavior and the resulting social welfare for different combinations of \( s, b \) and induced care \( x_I \).

For any price \( P \) charged by the firm and exercised care level \( x \), social welfare is given by

\[
W(x) = \alpha N(\theta_h - c(x) - p(x)H)I[P \leq \theta_h] + (1 - \alpha)N(\theta_l - c(x) - p(x)H)I[P \leq \theta_l]
\]

(1)

where \( I[\cdot] \) is the indicator function.\(^9\) If high-valuation consumers consume the product the social surplus per unit is \( \theta_h - c(x) - p(x)H \) (social benefit per unit minus the social cost per unit). Adding across all the high-type consumers and taking account of the fact that these consumers consume only if \( P \leq \theta_h \), gives the first term in the welfare function. The second term can be similarly explained. Now, if \( s, b \) and \( x_I \) are such that the firm’s full cost of production \( \tilde{C}(x_I; s, b) \) > \( \theta_h \), then there is no price at which both, the firm makes profit and consumers of at least one type are willing to purchase the product. In this case, therefore, nothing is produced and welfare is zero. If \( \theta_h < \tilde{C}(x_I; s, b) < \theta_l \), then the firm maximizes its profit with \( P = \theta_h \). It serves only the high valuation consumers and welfare is \( \alpha N(\theta_h - c(x_I) - p(x_I)H) \). If \( \tilde{C}(x_I; s, b) < \theta_l \), then the firm’s profit with price \( \theta_h \) is \( \pi(\theta_h) = \alpha N(\theta_h - \tilde{C}(x_I; s, b)) \) and with price \( \theta_l \), it is \( \pi(\theta_l) = N[\theta_l - \tilde{C}(x_I; s, b)] \). With price \( \theta_h \) per unit profit is higher but less units are sold; whereas, with price \( \theta_l \) per unit profit is lower but more units are sold because the low-valuation consumers also purchase the product. If the proportion of low-valuation consumers is small (i.e. \( \alpha \) is large), the gain in additional consumers is not enough to outweigh the effect of decreased price per unit and therefore the firm sells only to the high-valuation consumer. More precisely, if \( \alpha \gg (\theta_l - \tilde{C}(x_I; s, b))/\theta_h - \tilde{C}(x_I; s, b) \) the firm chooses price \( \theta_l \), which yields welfare \( \alpha N[\theta_h - c(x_I) - p(x_I)H] \); otherwise, it chooses \( \theta_l \) and the welfare is \( \alpha N[\theta_h] + (1 - \alpha)N[\theta_l] - N[c(x_I) + p(x_I)H] \). Summarizing, we get that for any given \( s, b \) and induced care \( x_I \), the welfare is

\[
W(x_I; s, b) = \alpha N(\theta_h - c(x_I) - p(x_I)H)I[\tilde{C}(x_I; s, b) < \theta_h] + (1 - \alpha)N[\theta_l - c(x_I) - p(x_I)H]I[\tilde{C}(x_I; s, b) < \theta_l]
\]

\[
- p(x_I)H]I[\tilde{C}(x_I; s, b) < \theta_l] \left[ \frac{\alpha \leq \theta_l - \tilde{C}(x_I; s, b)}{\theta_h - \tilde{C}(x_I; s, b)} \right]
\]

(2)

Using (2) and assumption (A.4), it can be shown that inducing any \( x \) for which \( c(x) + p(x)H > \theta_l \), is not optimal. For any such \( x \), welfare \( W(x) \leq \alpha N[\theta_h - c(x) - p(x)H] \). By setting \( s = x^* \) and \( b = 1 \) instead (which induces care level \( x^* \)) one can attain welfare \( W(x^*) \geq \alpha N[\theta_h - c(x^*) - p(x^*)H] > \alpha N[\theta_h - c(x) - p(x)H] \).\(^{10}\) Similarly, it can be shown that

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\(^9\) That \( I[\cdot] \) is an indicator function means, \( I[P \leq \theta_l] = 1 \) if \( P \leq \theta_l \) and zero otherwise.

\(^{10}\) With \( s = x^* \) and \( b = 1 \) the firm exercises \( x^* \), in which case the firm’s full marginal cost is \( c(x^*) \). Since this cost is smaller than \( \theta_h \), the firm will sell at least to the high-type consumer and hence, welfare will be at least \( \alpha N[\theta_h - c(x^*) - p(x^*)H] \). If it sells to both the types then welfare will be even higher (since by assumption (A.4), social benefit from consumption by a low-valuation consumer is higher than social cost when care is \( x^* \)). This explains the first inequality. The second inequality follows from the fact that \( x^* \) minimizes full social cost of production.
inducing any care level $x > x^*$ cannot be optimal. This leaves us with care levels $x$, which satisfy $x \leq x^*$ and $c(x) + p(x)H < \theta_i$. Proposition 1 below characterizes the optimal actual care level and the optimal due-care standard for inducing any $x \leq x^*$.

**Proposition 1.** In the absence of any legal error, (i) the best way to induce any care level $x \leq x^*$, is to set $b = 1$ and due-care standard $s$ at the same level as the desired care level. (ii) If $\alpha \geq \theta_i/\theta_h$ or $\alpha \leq (\theta_i - c(x^*))/(\theta_h - c(x^*))$ then the optimal actual care level $x_1^o = x^*$. For any $\alpha \in ((\theta_i - c(x^*))/(\theta_h - c(x^*)), (\theta_i/\theta_h))$, let $x_\alpha = c^{-1}((\theta_i - \alpha \theta_h)/(1 - \alpha))$. Then for $(1 - \alpha)[\theta_i - c(x_\alpha) - p(x_\alpha)H] > \alpha \{c(x_\alpha) + p(x_\alpha)H - (c(x^*) + p(x^*)H)\}$, the optimal actual care level $x_1^o = x_\alpha < x^*$, otherwise $x_1^o = x^*$.

The intuition behind why the due-care standard must be set equal to the desired care level in the absence of legal error is the following: for any desired and induced $x$, the higher is the firm’s per unit cost of production, the more likely it is that it will shut-down the low-valuation consumer and that welfare will be smaller (since for the optimal $x$, $c(x) + p(x)H < \theta_i$, it is desirable that low-valuation consumers consume the product). By setting $s = x$ and $b = 1$, we can make the firm’s expected liability cost zero, which results in the least possible full cost per unit produced. In the absence of legal error it is not possible to induce $x$ with any $s < x$, and any $s > x$ results in positive liability costs.

Note that even if liability costs become zero with $s = x$ and $b = 1$ and that penalty is not socially costly in equilibrium (it has no influence on price or welfare), it is not necessarily optimal to induce care $x^*$, the level that minimizes the expected social cost of production. A decrease in care level from $x^*$ decreases the firm’s cost, which can decrease the price and increase the number of consumers who can afford the product. When the resulting gain in consumer surplus, $(1 - \alpha)N[\theta_i - c(x) - p(x)H]$, exceeds the loss due to inefficient choice of care level, $\alpha N[(c(x) + p(x)H) - (c(x^*) + p(x^*)H)]$, then it is desirable to induce a lower care level. However, regardless of which care level is desired, it is optimal to set the due-care standard at that care level. This is not necessarily true once we allow for legal error.

4. Optimal location of the due-care standard when there is legal error

With legal error, for any due-care standard $s$ and actual care level $x$, the firm has to pay damages with probability $p(x)\xi(s - x)$ (with probability $p(x)$ there is an accident, given which it is found liable with probability $\xi(s - x)$). Hence for any $s$ and $b$, the firm chooses $x_I$ which minimizes

$$c(x) + p(x)\xi(s - x)bH. \quad (3)$$

This implies a full marginal cost to the firm of $\tilde{C}(x_I; s, b) = c(x_I) + p(x_I)\xi(s - x_I)bH$. Unlike in the benchmark case, now the likelihood of being found liable and hence, the firm’s full cost depends not only on whether $x_I$ is below or above $s$, but also on how much lower or higher it is. For any $\tilde{C}(x_I; s, b)$ the firm’s pricing decision and the consequent welfare can be determined exactly as in the previous section. Doing so we get for any $s, b$
and \( x_I \):

\[
W(x_I; s, b) = \alpha N(\theta_h - c(x_I) - p(x_I)H)[\tilde{C}(x_I; s, b) < \theta_h] + (1 - \alpha)N[\theta_l - c(x_I) - p(x_I)H][\tilde{C}(x_I; s, b) < \theta_l]I \left[ \alpha \leq \frac{\theta_l - \tilde{C}(x_I; s, b)}{\theta_h - \tilde{C}(x_I; s, b)} \right],
\]

which is the same expression as in (2), except that now full marginal cost is \( \tilde{C}(x_I; s, b) = c(x_I) + p(x_I)\xi(s - x_I)bH \).

Assuming that \( p(x)\xi(s - x) \) is convex in \( x \) (i.e. the likelihood of damage payment decreases at a decreasing rate with care), induced care \( x_I \) corresponding to any choice of \((s, b)\) is given by the first order condition (suppressing function arguments for brevity):

\[
c'(x_I) = -[p'\xi - \xi']bH. \tag{5}
\]

This condition says that the firm will choose that care level, where marginal gain from additional care (which is the marginal decrease in expected damages, \(-[p'\xi - \xi']bH\)) equals the marginal cost, \( c'(x_I) \). Solving for \( b \) from (5), to induce any given \( x \) with a standard \( s \), penalty multiplier \( b \) must satisfy

\[
b(s, x) = \frac{c'(x)}{[p(x)\xi'(s - x) - p'(x)\xi(s - x)]H}. \tag{6}
\]

Substituting this \( b \) in \( \tilde{C}(x; s, b) \), we get that for any \( x \) that is induced with a standard \( s \), the firm’s full marginal cost will be

\[
\tilde{C}(x; s) = c(x) + \frac{c'(x)}{\xi'(s - x)/\xi(s - x) - (p'(x)/p(x))}. \tag{7}
\]

As can be seen from the second term on the RHS of this equation, the firm’s expected liability cost depends on the due-care standard and increases (resp. decreases) with it when \((\xi'(s - x))/((\xi(s - x))\) is a decreasing (resp. increasing) function of \( s \). This suggests that given any desired care level, it may be possible to manipulate \( s \) to influence the expected liability cost. Whether \( s \) should be made more stringent or lenient than the desired care depends on the nature of legal error, and whether we want to decrease or increase the expected liability cost of inducing any desired care. Assuming that the objective is to minimize this cost, we investigate the optimal location of \( s \) for three different forms of legal error.\[^{11}\]

**Case 1** (only type-II error). Suppose that it is possible that a guilty firm is exonerated with some probability less than one, but an innocent firm is never found liable. This may happen, for example, when the standard of proof required by the court is so high that it is impossible to gather convincing evidence against an innocent firm. This implies \( \xi(s - x) = 0 \) for \( x \geq s \), and \( 0 < \xi(s - x) < 1 \) for \( x \leq s \). In this case welfare cannot be improved by making the due-care standard more lenient or stringent than the desired care. By setting these equal and with a large enough penalty multiplier, the firm can be induced to exercise the desired care with no expected liability cost in equilibrium. The intuition is straightforward. If the

\[^{11}\text{Proposition 2 proves that in our set-up welfare is maximized when the optimal actual care is induced with the least possible expected liability cost.}\]
standard equals the desired care then for any exercised care below it, the firm’s likelihood of being found liable is positive. By choosing a large enough multiplier we can ensure that the firm’s marginal gain from additional care is greater than its marginal cost, at any care level below the desired care. This induces the firm to exercise additional care until it reaches the desired care. Once the firm exercises the desired care, it is not found liable and therefore its expected liability cost becomes zero.12

**Case 2** (both type-I and type-II errors, but type-I error is small). Suppose that both type I and type II errors are possible, however, type-I error is small in the sense that if the firm exercises a sufficiently higher care than the due-care standard then it is not found liable. More precisely, let us say that type-I error is small and of size $K$, if with a care that is $K$ levels or more above the due-care standard the firm cannot be found liable, otherwise it may be found liable (i.e. $\xi(s-x) = 0$ for all $s \geq s + K$ and $0 < \xi(s-x) < 1$ for $x < s + K$). In this case if $x^{oo}$ is the desired care and $x^{oo} - K > \hat{s}$, where $\hat{s}$ is the minimum possible due-care standard, then the optimal due-care standard is exactly $K$ levels below $x^{oo}$. This due-care standard ($s = x^{oo} - K$), along with penalty multiplier $b = (c'(x^{oo}))/p\xi(-K)H$ induces $x^{oo}$ with no expected liability cost in equilibrium. The intuition is the same as above. With standard at $x^{oo} - K$, the firm’s likelihood of being found liable is positive for any exercised care below $x^{oo}$. With a large enough penalty multiplier it can be ensured that the firm’s marginal gain from additional care is greater than its marginal cost for any care below $x^{oo}$. Once the firm exercise care $x^{oo}$ (which is $K$ levels above the standard) it is not found liable and therefore its expected liability cost becomes zero. Unlike in case 1, expected liability cost cannot be made zero with $s=x^{oo}$, since the probability of erroneous conviction is positive when the due-care equals the exercised care.

If it is not possible to set due-care at $K$ levels below the desired care, say because it is smaller than $\hat{s}$, then the optimal due-care should be determined as in case 3 below.

**Case 3** (both type-I and type-II errors and both are large). Suppose both type-I and type-II errors are large in the sense that $0 < \xi(s-x) < 1$ for all $s-x$ and $s \geq \hat{s}$. That is, regardless of how much the actual care exceeds the due-care standard, the likelihood of being found liable remains positive. In this case the optimal due-care depends on whether $(\xi'(s-x))/(\xi(s-x))$ is a decreasing or an increasing function of $s$ (i.e. whether $\xi(\cdot)$ is log-concave or log-convex, respectively). If it is the former, then the due-care standard should be made as lenient as possible, that is optimal $s = \hat{s}$. Algebraically, this follows easily from the the fact that $(\xi'(s-x))/(\xi(s-x))$ is negatively related to the expected liability cost (see equation (7)). For a similar reason, if $(\xi'(s-x))/(\xi(s-x))$ increases with $s$ then the due-care standard should be made as stringent as possible, i.e. $s = X$.

Let us try to understand intuitively why a more lenient (resp. stringent) due-care standard improves welfare when $(\xi'(s-x))/(\xi(s-x))$ is a decreasing (resp. increasing) function of $s$. First, note that as the firm exercises more care, the likelihood of damage payment, $p(x)\xi(s-x)$, falls. Let us refer to this marginal decrease, $-(\partial [p(x)\xi(s-x)]/\partial x) = p\xi' - \xi p'$, as the ‘effectiveness of additional care’. Now suppose that a standard and a penalty multiplier induces some desired care and let us see what happens to the expected liability cost

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12 To see this formally, suppose $x^{oo}$ denotes the desired care. Then it can be checked that with $s=x^{oo}$ and $b = c'(x^{oo})/(p\xi'(0)H)$, $\partial C(x; s, b)/\partial x < 0$ for $x < x^{oo}$ and $\partial C(x; s, b)/\partial x > 0$ for $x \geq x^{oo}$. This implies that the firm will exercise care level $x^{oo}$ and since $\xi(x^{oo} - x^{oo}) = 0$, the firm’s expected liability cost will be zero.
when the due-care standard is increased and the penalty multiplier adjusted so that the firm sticks to the desired care. Given other things, an increase in \( s \) changes the effectiveness of additional care.\(^{13}\) Supposing that this effectiveness increases with the stringency of standard, other things equal, the firm’s marginal gain from additional care increases. Everything else equal this would increase the actual care exercised above the desired care. To prevent this, the penalty multiplier has to be decreased by the same percentage as the percentage increase in effectiveness of additional care. What impact does all of this have on the firm’s expected liability cost? Given other things, an increase in \( s \) also increases the likelihood of damage payment. Since the expected liability cost is proportional to the penalty multiplier times the likelihood of damage payment, if the percentage increase in latter is greater (smaller) than the percentage decrease in the former then the expected liability cost increases (decreases). The percentage decrease in penalty multiplier is chosen to be the same as percentage increase in effectiveness of additional care and therefore the impact on expected liability cost depends on whether an increase in \( s \) causes a greater percent increase in likelihood of damage payment \( (p\xi') \), or in effectiveness of additional care \( (p\xi - \xi p') \). As can be seen from these expressions, the percentage change in the former is the same as percentage change in \( \xi \), whereas, percentage change in the latter is some weighted average of percent changes in \( \xi(\cdot) \) and \( \xi'(\cdot) \). If \( \xi'/\xi \) is a decreasing function of \( s \), it means that percent change in \( \xi' \) is smaller than percent change in \( \xi \), which implies that percentage increase in likelihood of damage payment is greater than that in effectiveness of additional care \( (p\xi' - \xi p') \). Hence, in this case the expected liability cost rises with \( s \). Alternatively, it falls as \( s \) decreases and this calls for making \( s \) more lenient when \( \xi(s - x) \) is log-concave. Similar argument shows why \( s \) should be made more stringent when \( \xi(\cdot) \) is log-convex.

It may be difficult to determine whether in reality \( \xi(\cdot) \) is log-concave or log-convex. For some of the standard functions one can think of, it is log-concave. For example, function \( \xi(\cdot) \) assumed in the seminal Craswell and Calfee (1986) paper is log-concave.\(^{14}\) Also, it can be shown that any function of the form \( \xi(s - x) = (a_1(s - x + K)^n + a_2(s - x + K)^{n+1} + a_3(s - x + K)^{n+2})/M \) where, \( a_k \geq 0 \) (\( k = 1, 2, 3 \)), \( n \geq 1 \) and \( K \) and \( M \) are positive constants chosen to ensure that \( 0 < \xi(s - x) < 1 \), is log-concave. This family of functions can used to approximate various shapes by changing the parameter values. Linear and quadratic \( \xi(\cdot) \) are special cases of this family of functions (see Bhole, 2005).

We summarize the findings in the above cases in the following proposition. We also prove that in our set-up, the welfare is maximized when the optimal actual care is induced with the least expected liability cost.

\[^{13}\] There are two (potentially opposite) effects of a more stringent standard. First, since the firm is more likely to be found liable given an accident (see assumption (A.3)) any decrease in likelihood of accident due to additional care is more effective (i.e., \( -\xi p' \) is higher). Second, a more stringent standard means a greater difference between the due-care and the desired care. If the conditional likelihood of being found liable, \( \xi(s - x) \), increases at an increasing (respectively, decreasing) rate, then with a more stringent standard any additional care is more (respectively, less) effective in decreasing the likelihood of being found liable; i.e., \( p\xi' \) is larger (resp. smaller).

\(^{14}\) Craswell and Calfee (1986) do not explicitly specify any probability of conviction function. However, their assumption that the court’s interpretation of standard is normally distributed around a known mean, implies probability of conviction function \( \xi(s - x) = \int_{-\infty}^{s} \phi(x) \, dx \) where, \( \phi(\cdot) \) is the normal density function with mean \( x \) and some variance \( \sigma^2 \), which is log-concave. See also footnote 2.
Proposition 2.

(a) Denote by \( x^o_0 \) the optimal actual care in the presence of legal error. Then welfare is maximized when \( s \) is chosen to minimize the firm’s full marginal cost, \( \tilde{C}(x^o_l, s) \).

(b) If there is only type-II error, then optimal due-care standard is \( s = x^o_0 \), the same as the desired care.

(c) If there are both type I and type II errors, but type I error is small and of size \( K \) (see Case 2 above) then optimal due-care standard is \( x^o_0 - K \) provided \( x^o_0 - K \geq \hat{s} \), i.e. due-care standard should be more lenient than the desired care. If \( x^o_0 - K < \hat{s} \), then due-care should be determined as in part (d).

(d) If there are both type I and type-II errors and they are large (see case 3 above) then optimal due-care standard is \( s = \hat{s} \) (resp. \( s = X \)) if \( \xi(\cdot) \) is log-concave (resp. log-convex). That is due-care should be as lenient (resp. stringent) as possible relative to \( x^o_0 \).

The intuition behind part (a) is straightforward. The smaller is the firm’s marginal cost the more likely it is that price will be smaller, and therefore more likely it is that the firm will sell to both types of consumers, which is desirable in our setting. The proposition suggests that when \( \xi(s - x) \) is log-concave, social welfare can be improved by making the due-care standard more lenient than the desired care and adjusting the size of penalty multiplier such that the firm overcomplies and exercises the desired care. This suggests a possible explanation for why sometimes standards appear to be too lax and are seen as prescribing insufficient care-level. This explanation for laxity of standards is different from that offered by Boyer et al. (2000). In their analysis, the laxity of standards follows from the laxity of desired care. Instead, what we propose is that lowering the due-care standard may improve welfare even if the desired (and induced) care-level is left unchanged.

The foregoing analysis also suggests why in many cases the due-care standard may not be very different from the desired care. For any given desired care, adjustment in the standard requires adjustment in the penalty multiplier. Exogenous restrictions on the latter either due to wealth constraints or legal constraints, may restrict the extent to which the due-care standard can be manipulated to improve welfare.

The particular adjustments in the due-care standard suggested in Proposition 2 depend crucially on the fact that welfare is negatively related to the firm’s liability cost (which definitely seems to be the case in medical care\(^{15}\). If welfare is positively related to the firm’s cost then optimal location would be different from that suggested above. But in either case, setting the due-care standard equal to the desired care is likely to be optimal only in very special cases and generally welfare may be improved by manipulating it away from the desired care and adjusting the penalty multiplier accordingly.

As far as optimal actual care in the presence of legal error, \( x^o_0 \), is concerned, there are no surprising results. Proposition 3 in Appendix A characterizes \( x^o_0 \) and compares it with optimal care in the absence of legal error, \( x^*_l \). As in Section 3 it is shown that \( x^o_0 \leq x^* \).

\(^{15}\) See, for example, the report prepared by U.S. Department of Health and Human Services (2002). It cites several examples where increased malpractice insurance has either caused doctors to relocate their services or give it up altogether. To quote one example “At Frankford Hospital’s three facilities in Northeast Philadelphia and Bucks County, all twelve active orthopedic surgeons decided to lay down their scalpels after their malpractice rates nearly doubled to 106,000 each for 2001.”
Table 1
Different \((s, b)\) combinations that induce care \(x = 7\)

<table>
<thead>
<tr>
<th>(s)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.62</td>
</tr>
<tr>
<td>2</td>
<td>1.39</td>
</tr>
<tr>
<td>3</td>
<td>1.22</td>
</tr>
<tr>
<td>4</td>
<td>1.08</td>
</tr>
<tr>
<td>5</td>
<td>0.97</td>
</tr>
<tr>
<td>6</td>
<td>0.88</td>
</tr>
<tr>
<td>7</td>
<td>0.81</td>
</tr>
<tr>
<td>8</td>
<td>0.75</td>
</tr>
<tr>
<td>9</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Also, \(x_I^{oo}\) can be either greater or smaller than \(x_I^0\). This illustrates the well-known point in the moral hazard literature that implemented level of effort with hidden action (or care) can either be smaller or greater than that implemented under full-information (see Mas-Colell et al., 1995).

Before we conclude, we illustrate with a simple numerical example how \(s\) and \(b\) can be manipulated to reduce the firm’s expected liability cost while inducing it to undertake a given desired care level. Suppose that the set of possible care levels is given by \(\chi = [0, 9]\); the minimum due-care standard, \(\delta = 1\); \(p(x) = 0.9 - 0.09x\); \(\xi(s - x) = 0.05(s - x + 9)\) for \(s \geq 3\); \(c(x) = 0.5x^2\); \(H = 160\); the desired care level is \(x = 7\). Using (6), we get that for any standard \(s \in [1, 9]\) and penalty multiplier \(b(s, x)|_{x = 7} = x/((0.9 - 0.09x) \times 0.05 + 0.09 \times 0.05 \times (s - x + 9)) \times 160 = 4.375/(1.35 + 0.45(2 + s))\) the firm exercises the desired care, \(x = 7\). Table 1 above shows these standard-multiplier combinations and Fig. 1 shows the firm’s expected liability cost when care level \(x = 7\) is induced with different standard-multiplier combinations.

![Fig. 1. The firm’s expected liability cost when care level \(x = 7\) is induced with different standard-multiplier combinations.](image-url)
expected liability cost corresponding to these different combinations. Since the assumed $\xi(\cdot)$ is log-concave we see that the larger the due-care standard the larger is the expected liability cost. Therefore, the due-care standard should be made as lenient as possible; which in this case suggests setting $s = 1$.

5. Conclusion

It is not difficult to find examples in health care and construction services where cost and provision have been adversely affected by liability considerations. The same holds true in case of many products (see for example Viscusi, 1991). Therefore, it seems important to find ways that induce firms to take appropriate care but at the same time reduce their expected liability costs. This paper suggests one possible solution when there is a possibility of legal error. When there is no legal error, a firm’s expected liability cost can be made zero by setting the due-care standard equal to the desired care and the penalty equal to the harm. However, when there is legal error, a given care may be induced more efficiently by making the due-care standard either more lenient or stringent relative to the desired care level (depending on how the firm’s likelihood of being found liable changes with the difference between the due-care and the exercised care) and adjusting the penalty-multiplier accordingly. If there are exogenous restrictions on penalty multiplier, say due to wealth constraints or any other reasons, then that may restrict the extent to which due-care standard can be manipulated and hence the amount by which expected liability cost can be decreased.

An interesting extension of the model would be to include litigation costs. It has been assumed in this paper that the injured party (or the regulatory agency) brings lawsuit following every accident, which implicitly assumes a zero or a very small cost of litigation relative to the expected award. If litigation costs are substantial then plaintiff’s decision to bring a lawsuit may be affected by leniency of the due-care standard (as that would imply a smaller likelihood of conviction for any given desired and exercised care level), which may in turn affect a firm’s incentive to take care. These incentives may then have to be corrected by readjusting the due-care standard and/or the penalty multiplier. It will be interesting to see whether manipulation of the due-care standard away from the desired care level can increase welfare in these circumstances.

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16 See U.S. Department of Health and Human Services (2002) for many such examples. To quote one example from this report, most cities with population under 20,000 in Mississippi no longer have doctors who deliver babies. Doctors want to or have relocated their practice because of the cost of insurance.
Appendix A

Proof of Proposition 1.

(i) For any standard \( s \) and \( b > 0 \) that induces care level \( x \leq x^* \), welfare is given by (2). Note that \( \theta_l - \tilde{C}(x; s, b)/\theta_h - \tilde{C}(x; s, b) \) is decreasing in \( \tilde{C}(x; s, b) \), and hence the smaller is \( \tilde{C}(x; s, b) \) the greater is the welfare. Now any care level \( x \leq x^* \) is induced with least possible \( \tilde{C}(x; s, b) \) when \( s = x \) and \( b = 1 \) (with this standard and penalty function, \( \tilde{C}(x; s, b) = c(x) \)). Hence, it is optimal to set \( b = 1 \) and the due-care standard at the same level as the desired care level.

(ii) If \( \alpha > \theta_l/\theta_h \), then for any optimally induced care level \( x \) less than \( x^* \) (i.e. induced with \( s = x \) and \( b = 1 \)), welfare is at most \( \alpha N(\theta_h - c(x) - p(x)H) \). Clearly, this is smaller than welfare when \( x^* \) is optimally induced, which is \( \alpha N(\theta_h - c(x^*) - p(x^*)H) \). Hence, \( x^* \) is the optimal care level. Similarly, for \( \alpha \leq \theta_l - c(x^*)/\theta_l - c(x^*) \), welfare with any \( x \) is at most \( \alpha N(\theta_h - c(x) - p(x)H) + (1 - \alpha) N[\theta_l - c(x) - p(x)H] \), which cannot be larger than \( \alpha N(\theta_h - c(x^*) - p(x^*)H) + (1 - \alpha) N[\theta_l - c(x^*) - p(x^*)H] \). When \( \alpha \in (\theta_l - c(x^*)/\theta_l - c(x^*), \theta_l/\theta_h) \), then by definition, at \( x_\alpha \) we have \( \alpha = \theta_l - c(x_\alpha)/\theta_l - c(x_\alpha) \) and welfare equal to \( \alpha N[\theta_h] + (1 - \alpha) N[\theta_l] - N[c(x_\alpha) + p(x_\alpha)H] \). If \( x^* \) is induced then the firm will charge price \( \theta_h \) (since, \( \alpha > \theta_l - c(x^*)/\theta_l - c(x^*) \)) and welfare will be \( \alpha N[\theta_h - c(x^*) - p(x^*)H] \). If \( (1 - \alpha) [\theta_l - c(x_\alpha) - p(x_\alpha)H] \geq \alpha \{(c(x_\alpha) + p(x_\alpha)H) - (c(x^*) + p(x^*)H) \} \) then welfare is higher with \( x_\alpha \), otherwise welfare is higher when \( x^* \) is induced.

Proof of Proposition 2. Part (a): We show that welfare is maximized when \( s \) is set such that \( \tilde{C}(x_{I}^{oo}, s) \) is minimized. First, suppose that \( x_{I}^{oo} \) is such that \( c(x_{I}^{oo}) + p(x_{I}^{oo}) \) \( H < \theta_l \). In this case it is socially optimal that both types consume the product. Now, the higher is \( \tilde{C}(\cdot) \), the smaller is \( \theta_l - \tilde{C}(x_{I}^{oo}, s)/\theta_l - \tilde{C}(x_{I}^{oo}, s) \). Using (4) this implies that, the higher is \( \tilde{C}(\cdot) \) the more likely it will be that low-valuation consumers will be shut-down and therefore that welfare will be lower. Hence, \( s \) should be chosen to minimize \( \tilde{C}(x_{I}^{oo}, s) \).

Even if \( c(x_{I}^{oo}) + p(x_{I}^{oo})H > \theta_l \), then too there is no harm in choosing \( s \) that minimizes \( \tilde{C}(x_{I}^{oo}, s) \). One need not worry about a situation where minimizing \( \tilde{C}(x_{I}^{oo}, s) \) with respect to \( s \) results in \( \tilde{C}(x_{I}^{oo}, s) < \theta_l \) and there is an overprovision of the service. Both \( c(x_{I}^{oo}) + p(x_{I}^{oo})H > \theta_l \) and \( \tilde{C}(x_{I}^{oo}, s) < \theta_l \) cannot hold for any \( s \), if \( x_{I}^{oo} \) is the optimal induced care. As will be shown in the proof of Proposition 3, \( x_{I}^{oo} \leq x^* \). Hence, if both the above conditions happen to hold then induced care, \( x_I \) can be increased above \( x_{I}^{oo} \) until either \( \tilde{C}(x_{I}; s) \geq \theta_l \) or \( c(x_{I}^{oo}) + p(x_{I}^{oo})H \leq \theta_l \). From (A.4) we know that at least one of these happens for some \( x_I \leq x^* \). In either case, welfare will be higher with the new induced care contradicting the fact that \( x_{I}^{oo} \) was optimal induced care. From (7) it is clear that if \( \xi(\cdot) \) is log-concave then to make \( \tilde{C}(x_{I}^{oo}, s) \) smaller the due-care standard should be made more lenient.

Parts (b), (c) and (d): Follow easily from the discussion in Cases 1–3.

17 In proof of Proposition 3 it is shown that \( \tilde{C}(x_{I}; s) \) is increasing in \( x_I \).
**Proposition 3.** Assuming $\tilde{C}(x^*; s) < \theta_h$ for some $s$, then (a) The optimal actual care, $x_I^{oo}$ when there is legal error will satisfy $x_I^{oo} \leq x^*$ and $c(x_I^{oo}) + p(x_I^{oo})H < \theta_l$. (b) If $\alpha \geq \theta_l/\theta_h$ or $\alpha \leq (\theta_l - \tilde{C}(x^*; s))/(\theta_l - \tilde{C}(x^*; s))$ where $s$ is optimally chosen in accordance with **Proposition 2**, then $x_I^{oo} = x^*$ (and consequently, $x_I^{oo} = x_0^o$). (c) For $\alpha \in [(\theta_l - \tilde{C}(x^*; s))/(\theta_l - \tilde{C}(x^*; s)), \theta_l/\theta_h]$, if there exists $\tilde{x}_\alpha$ such that $(\theta_l - \tilde{C}(\tilde{x}_\alpha; s))/(\theta_l - \tilde{C}(\tilde{x}_\alpha; s)) = \alpha$, then

$$\left(1 - \alpha\right)\left[\theta_l - c(\tilde{x}_\alpha) - p(\tilde{x}_\alpha)\right] > \alpha\left\{c(\tilde{x}_\alpha) + p(\tilde{x}_\alpha)H - (c(x^*) + p(x^*)H)\right\},$$

then $x_I^{oo} = \tilde{x}_\alpha < x^*$; otherwise $x_I^{oo} = x^*$. Therefore, if (8) holds then $x_I^{oo} > x_I^{oo}$ otherwise $x_I^{oo} \leq x_I^{oo}$.

**Proof.** Part (a): Our assumption that $p(x)\xi(s - x)$ is a convex function of $x$ implies that $\tilde{C}(x; s)$ is an increasing function of $x$ for any given $s$. To see this, note that for any $x$ induced by $(s, b)$, we must have from (5) $c' + b[p\xi - p\xi']H = 0$. Convexity of $p\xi$ implies that to induce $\tilde{x} > x$, we have to increase $b$ to some value say $\tilde{b}$. Thus, $\tilde{C}(x; s, b) = c(x) + p(x)\xi(s - x)\tilde{b}H \leq c(\tilde{x}) + p(\tilde{x})\xi(s - \tilde{x})\tilde{b}H < c(\tilde{x}) + p(\tilde{x})\xi(s - \tilde{x})\tilde{b}H = \tilde{C}(\tilde{x}; s)$. Weak inequality follows from the fact that $x$ minimizes the firm’s full marginal cost of production given (s, b) and the strict inequality follows from the fact that $\tilde{b} > b$. Using (4) and the fact that $\tilde{C}(x; s)$ is increasing in $x$, it is clear that the welfare with any $x > x^*$ is smaller than welfare at $x^*$. Therefore, $x^{oo} < x^*$.

Now we show that optimal $x^{oo}$ will satisfy $c(x^{oo}) + p(x^{oo})H < \theta_l$. Suppose $x^{oo}$ is optimal care and $c(x^{oo}) + p(x^{oo})H > \theta_l$. Then from (4) it is clear that $W(x^{oo}; s) = \alpha N[\theta_h - c(x^{oo}) - p(x^{oo})H]$. By inducing $x^*$ instead with some $x'$ we can attain welfare $W(x^*; s') = \alpha N[\theta_h - c(x^*) + p(x^*)H] > \alpha N[\theta_h - c(x^{oo}) - p(x^{oo})H] = W(x^{oo}; s)$. The fact that $x^*$ be induced with some $s'$ follows from our assumption that $\tilde{C}(x^*, s) < \theta_h$ for some $s$. In case this assumption is not satisfied then (because $\tilde{C}(x^*, s)$ is increasing in $x$) we may have to induce $x_I^{oo}$ for which $c(x^{oo}) + p(x^{oo})H > \theta_l$.

Parts (b) and (c): Proof of part (b) and most of part (c) is similar to that of part (ii) of **Proposition 1**, except that $c(x)$ is replaced by $\tilde{C}(x; s)$ and $x_\alpha$ is replaced by $\tilde{x}_\alpha$. For comparison between $x^*$ and $x^{oo}$ note the following. If $\alpha \in \{(\theta_l - \tilde{C}(x^*; s))/(\theta_l - \tilde{C}(x^*; s)), \theta_l/\theta_h\}$ and (8) holds then $x^{oo} = \tilde{x}_\alpha$. Whereas (from **Proposition 1**), for $\alpha \in \{(\theta_l - \tilde{C}(x^*; s))/(\theta_l - \tilde{C}(x^*; s)), (\theta_l - c(x^*))/(\theta_l - \tilde{C}(x^*; s))\}$, $x^* = x^*$ and for $\alpha \in \{(\theta_l - c(x^*))/(\theta_l - \tilde{C}(x^*; s)), \theta_l/\theta_h\}$, $x^* = x_\alpha$ or $x = x^*$. Since $\tilde{x}_\alpha < x_\alpha$, it follows that $x^{oo} < x^*$. If (8) does not hold then $x^{oo} = x^*$. But even if (8) does not hold, it is still possible that

$$(1 - \alpha)[\theta_l - c(x_\alpha) - p(x_\alpha)H] \geq \alpha\left\{c(x_\alpha) + p(x_\alpha)H - (c(x^*) + p(x^*)H)\right\},$$

in which case $x^{oo} = x_\alpha < x^* = x^{oo}$. \(\square\)

**References**


