Compositions of Integers

Ralph Grimaldi
Rose Hulman Institute of Technology
ralph.grimaldi@rose-hulman.edu
Terre Haute, IN 47803

A composition of a positive integer n is an ordered sum of positive summands whose total is n. When the summands are restricted to the odd integers one finds that the number of resulting compositions is counted by the Fibonacci numbers. This is also the case when the summands must all exceed 1. If the last summand is required to be odd the number of resulting compositions is counted by the Jacobstahl numbers. These results and related ideas will be examined in this talk.

Pebbles, Trees and Rigid Graphs

Ruth Haas
Department of Mathematics, Smith College
rhaas@math.smith.edu
Northampton, MA 01063

The Pebble Game was developed as an algorithm for determining whether a graph is rigid. In this talk we give some new variations of the pebble game that can be used for deciding if a multi-graph is the union of k edge-disjoint spanning trees or satisfies rigidity type conditions. Along the way we discuss several old and new characterizations of arboricity and rigidity.
**On the Most Wanted Folkman Graph**

Stanislaw Radziszowski

*Rochester Institute of Technology*

*spr@cs.rit.edu*

*Rochester, NY 14623*

We discuss a branch of Ramsey theory concerning Folkman graphs and numbers. We write $G \rightarrow (a_1, \cdots, a_k;p)^c$ if for every edge $k$-coloring of an undirected simple graph $G$ not containing $K_p$, a monochromatic $K_{a_i}$ is forced in color $i$ for some $i \in \{1, \cdots, k\}$. The edge Folkman number is defined as

$$F_e(a_1, \cdots, a_k;p) = \min\{|V(G)| : (a_1, \cdots, a_k;p)^c\}.$$  

Folkman showed in 1970 that this number exists for $p > \max(a_1,\ldots,a_k) \cdot F_e(3,3;4)$ involves the smallest parameters for which the problem is open, namely the question, “What is the smallest order $N$ of a $K_4$-free graph, for which any edge 2-coloring must contain at least one monochromatic triangle?” It is known that $16 < N$ (an easy bound), and it is known through a probabilistic proof by Spencer that $N < 3 \cdot 10^9$. We suspect that $N < 127$.

This talk will present the background, overview some related problems, discuss the difficulties in obtaining better bounds on $N$, and give some computational evidence why it is very likely that $N < 100$.

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**A Combinatorial Approach to Key Predistribution for Distributed Sensor Networks**

Doug Stinson

*University of Waterloo*

*dstinson@caacr.math.uwaterloo.ca*

*Waterloo, Ontario, Canada N2L 3G1*

In this talk, we discuss the use of certain combinatorial set systems in the design of deterministic key predistribution schemes for distributed sensor networks. We concentrate on analyzing the connectivity and resilience of the resulting distributed sensor networks. Motivated by this application, we introduce a class of combinatorial designs which we term "common intersection designs" (or, CIDs). Several characterizations of "optimal" CIDs are given, which relate to other combinatorial structures such as group-divisible designs, generalized quadrangles and strongly regular graphs. (This talk is based on joint work with Jooyoung Lee.)
Generalized Kirkman Designs

Walter Wallis
Department of Mathematics, Southern Illinois University
Carbondale, IL 62901-4408
wdwallis@rocketmail.com

A schoolmistress has 15 girl pupils and she wishes to take them on a daily walk. The girls are to walk in five rows of three girls each. It is required that no two girls should walk in the same row more than once per week.
More generally, Kirkman asked the same question when the class size is an odd multiple of 3. Notice that this is actually a packing problem. But the numbers are such that it is simultaneously a covering problem.
What if the class size is not an odd multiple of 3? We could ask for a set of daily walks in which as many pairs walk together as is possible without repetitions (called a Kirkman packing design) or one where every pair must walk together at least once and the number of repetitions is minimized (a Kirkman covering design). And maybe one row of 2 or 4 girls must be allowed. These and other generalizations will be discussed.

Matching Covered Graphs

Earl Glen Whitehead, Jr.
Department of Mathematics, University of Pittsburgh
egw1@pitt.edu
Pittsburgh, PA 15260

A perfect matching of a graph G is a spanning subgraph of G consisting entirely of independent edges. G is said to be matching covered if for every edge e in G, there is a perfect matching of G that contains e. Various methods of determining which graphs are matching covered will be discussed.
We will consider the following types of graphs – complete tripartite graphs, mesh graphs, generalized theta graphs, cubic graphs, and cages. (This research is joint work with Kimberly Jordan Burch.)

What is Probability?

Peter Winkler
Dartmouth College
peter.winkler@dartmouth.edu
Hanover, NH 03755-3551

An exploration of the uses and misuses of probability in topics ranging from gambling and medicine to the likelihood of intelligent life elsewhere in the universe.
Contributed Talks

Tiling with 4 × 6 and 5 × 7 Rectangles

Rachell Ashley*, Aisosa Ayela-Uwangue*, Frances Cabrera*
Carol Callesano*, Darren A. Narayan, and Allen J. Schwenk
Department of Mathematics & Statistics, Rochester Institute of Technology
Rochester, NY 14623
dansma@rit.edu

We consider the problem of tiling large rectangles using smaller rectangles with the prescribed dimensions 4 × 6 and 5 × 7. Narayan and Schwenk showed in 2002 that all rectangles with length and width at least 34 can be partitioned into 4 × 6 and 5 × 7 rectangles. However the case involving rectangles with a dimension less than or equal to 33 is still unsolved. Our goal is to determine a definitive list of which rectangles can be tiled using 4 × 6 and 5 × 7 tiles and which rectangles cannot. Currently, we have reached a definitive conclusion for all but a finite number of cases.

Stochastic Population Model Based Extreme Competition for Habitat and its Asymptotic Approximation

Jemal Mohammed-Awel
Department of Mathematics, ACSU
Buffalo, NY, 14260-2900
jemalm@acsu.buffalo.edu

We developed stochastic population model using a discrete probability distribution which involves String number of the second kind. We computed the moments, asymptotic mean, asymptotic variance, and the coefficient of variations of the distribution. In order to run stochastic simulations using this probability model, we must be able effectively to draw deviation from the distribution, which we found it to be computationally expensive. We proved that under some conditions the probability distribution can approximated by Gaussian distribution with the same mean and variance which make the simulation much faster.
**Nordstrom-Robinson Code is the Image of an LT Code**

Ruben Aydinyan  
*Department of Mathematics, Central Michigan University*  
*Mount Pleasant, Michigan 48858*  
aydin1r@cmich.edu

Quaternary codes have become of major interest with the discovery that several nonlinear codes, such as those found by Nordstrom-Robinson, Kerdock, Preparata, Goethals, and Delsarte-Goethals, are the binary images of linear codes over $\mathbb{Z}_4$. To get a binary code from a $\mathbb{Z}_4$ code, one uses the Gray map, defined by $\Phi: \mathbb{Z}_4 \rightarrow \mathbb{Z}_2^2$: $0 \mapsto 00, 1 \mapsto 01, 2 \mapsto 11, 3 \mapsto 10$. It extends naturally to a map $\Phi: \mathbb{Z}_4^n \rightarrow \mathbb{Z}_2^{2n}$. This map is an isometry between $\mathbb{Z}_4^n$ and $\mathbb{Z}_2^{2n}$ if we consider $\mathbb{Z}_4^n$ with the Lee metric and $\mathbb{Z}_2^{2n}$ with the Hamming metric. The Nordstrom-Robinson code is shown here to be the binary image of the linear $[8,4,6]_4$ LT code under the Gray map.

**On $k$-Wise Set-Intersections**

Weiting Cao*, Kyung-Won Hwang, and Douglas B. West  
*Department of Mathematics, University of Illinois*  
*Urbana, IL 61801*  
wcao1@uiuc.edu

Let $p$ be a prime, and let $L$ be a set of $s$ congruence classes modulo $p$. Let $H$ be a family of subsets of $[n]$ such that the size modulo $p$ of each member of $H$ is not in $L$, but the size modulo $p$ of every intersection of $k$ distinct members of $H$ is in $L$. We prove that $|H| \leq (k - 1) \sum_{i=0}^{k-1} n^i$, improving the bound due to Grolmusz and Sudakov.

**Fractal Properties of the Matrix for the Cups and Stones Counting Problem**

Joaquin Carbonara* and David Ettestad*  
*Buffalo State College*  
*1300 Elmwood Av, Buffalo NY*  
CARBONJO@BuffaloState.edu

In [*Adv. in Appl. Math.* 21, 405-423 (1998)] Carbonara and Green study a new and interesting family of integer sequences $S_{k,\sigma}$, for $\sigma = 1$, motivated by a counting problem first posed by Barry Cipra [Math. Mag. 65(1) (1992), 56] and still open in the case of full generality ( [Cipra and others *Math. Mag.* 66(1) (1993), 58-59] and to the best knowledge of the authors). In addition to the enumerative combinatorics result, in that paper they give evidence that Sierpinski's Gasket shows up in the cases when $k = 2^n + 1$ in the matrix representation of the counting problem. The counting problem, when represented as a matrix can be viewed as a finite cellular automaton. In our current work, we show that the cellular automaton has a decomposition with fractal characteristics and dimension similar to Sierpinski’s Gasket.
The Core of a Cartesian Product Graph with \( n \) Many Odd-Angulated Factors

Zhongyuan Che* and Karen L. Collins
Department of Mathematics, Penn State Beaver Campus
Monaca, PA 15061-2799
zcc10@psu.edu

A graph homomorphism between graphs \( G \) and \( H \) is a map from \( V(G) \) to \( V(H) \) that preserves adjacency property. A graph \( G \) is called a core if there is no graph homomorphism from \( G \) to any its proper induced subgraphs. Let \( G \) be a connected graph with odd girth \( 2k + 1 \). Then \( G \) is strongly \((2k+1)\)-angulated if any two vertices of \( G \) are connected by a sequence of \((2k+1)\)-cycles with consecutive cycles sharing at least one edge. We show that if \( G \) is a strongly \((2k+1)\)-angulated core, then either \( G^n \) is a core for all positive integers \( n \), or the core of \( G^n \) is \( G \) for all positive integers \( n \) and \( G \) is homomorphically equivalent to a normal Cayley graph. We then apply the result to construct Cartesian product cores with factors from well-known classes of graphs.

On Super Edge-Magic Graphs that are Weak Magic

Dharam V. Chopra* and Sin-Min Lee
Department of Mathematics & Statistics, Wichita State University
Wichita, KS 67260-0033
dharam.chopra@wichita.edu

In this paper we consider graphs with no loops. A graph \( G(V, E) \), with \( |V| = p \) and \( |E| = q \), is called total edge magic if there exists a bijection \( f : G \to \{1, 2, \cdots, p+q\} \) such that for each edge \( e \) joining vertices \( u, v \) we have \( f(u) + f(v) + f(e) = c \) (a constant).

We denote by \( M(G) \) the set of all total edge-magic labelings. We state conditions for a magic graph to be strong and weak which involve the use of \( M(G) \). If \( f : G \to \{1, 2, \cdots, p\} \), then a total edge-magic graph is called a super edge-magic graph. Here we are concered with identifying the kind of super edge-magic graphs which are weak magic.

Decomposition of \( K_{m,n} \) into 4-cycles and 2t-cycles

Chao-Chih Chou* and Chin-Mei Fu
St. John’s University
Tamsui, Taipei Shien, Taiwan, ROC
chaomail.sju.edu.tw

In this paper we show the necessary conditions for the decomposition of the complete bipartite graphs \( K_{2m,2n} \) and \( K_{2n+1,2n+1} - F \) into cycles of two different lengths 4 and 2t, \( t > 2 \), where \( F \) is a 1-factor of \( K_{2n+1,2n+1} \). After that we prove that the results are true for \( t = 5 \) and 6.
On the Existence of Nash Equilibria in Power Based Flow Control for Virtual Circuits

Ping-Tsai Chung* and Richard Van Slyke
Department of Computer Science, Long Island University
Brooklyn, New York 11201
pchung@liu.edu

We study a power-based flow control algorithm for virtual circuit flow control in a decentralized network. This algorithm is based on a greedy heuristic. Each virtual circuit (or user) iteratively optimizes its individual performance measure, power, by adjusting its message rate to achieve an ideal tradeoff point between high throughput and low delay. Our work focuses on individual (or user) optimization as opposed to system optimization. Convergence analysis are based on a game theoretic formulation. Under this formulation, the existence of a Nash equilibrium point of the power based flow control for an arbitrary network configuration is shown. Furthermore, we discuss Nash equilibria for generalized flow control on telecommunication networks and establish properties of the optimal rate chosen by the greedy algorithm.

A Family of Comma-Free Codes with Even Word-Length

L. J. Cummings
Department of Mathematics, University of Waterloo
Waterloo, Ontario, Canada, N2L 3C5
ljcummings@math.uwaterloo.ca

Comma-free codes do not contain overlaps of codewords and are used to determine synchronization in sequential transmission. The Witt function is a well-known general upper bound for the number of codewords in any comma-free block code with codewords of fixed length \( n \) over an arbitrary finite alphabet with \( \sigma \) letters. It has been known for some time that this upper bound is attained for all odd \( n > 1 \) and there is an elegant algorithm for construction due to Schlotz. The Witt function is not an effective upper bound when \( n \) is even and it is a long-standing open problem to find one. We construct for each even \( n > 1 \) and alphabet of size \( \sigma \) a family of \( \frac{n\sigma^2}{2} \) comma-free codes with \( r\frac{\sigma^2}{2}(\frac{n}{2}-r)^{\sigma^2-\lfloor \frac{\sigma^2}{2} \rfloor} \) codewords where \( 1 \leq r < \frac{n}{2} \) and conjecture the upper bound in each case could be the cardinality of one of the constructed codes.
Rank Numbers for k-Caterpillars

Patrick Curran* and Darren A. Narayan
Department of Mathematics & Statistics, Rochester Institute of Technology
Rochester, NY 14623
pmc6047@rit.edu

A k-ranking of a graph is a labeling of the vertices with integers such that for any pair of vertices with the same label contains a vertex with a larger label. A k-ranking is minimal if reducing any label larger than 1 violates the described ranking property. The rank number of G is the smallest k such that G has a minimal k-ranking. A k-caterpillar is a tree that consists of a spine, and a set of paths on k vertices or less having endpoints on the spine. We present new results involving rank numbers for k-caterpillars.

On the Existence of Some Combinatorial Arrays

Dharam Chopra and R. Dios*
Department of Mathematical Sciences, New Jersey Institute of Technology
Newark, New Jersey 07102
dios@adm.njit.edu

Some numerical arrays with a specified combinatorial structure must satisfy certain conditions mandated by that structure. These conditions formulate existence criteria in the form of inequalities that involve thoughtful computation. We focus upon balanced and orthogonal arrays for which some techniques will be presented along with some interesting examples.

On Maximal Instantaneous Codes and Binary Trees

Stephan Foldes* and Navin M. Singhi
Institute of Mathematics, Tampere University of Technology
PL 553, 33101 Tampere, Finland
stephan.foldes@tut.fi

We consider topologically sorted path-length sequences of binary trees, which correspond to codeword length sequences of binary maximal instantaneous codes taken in the lexicographic order of codewords. As opposed to unordered multi-sets of lengths, characterized by the Kraft equality, such length sequences completely determine the topological trees and binary codes to which they correspond. Those ordered sequences of integers that represent binary topological trees and maximal instantaneous codes are characterized by a family of Kraft-type equalities.
Independent Sets and Leaves: Some Connections

Bert Hartnell*, Rommel Barbosa, and Doug Rall
Department of Mathematics & Computing Science, Saint Mary’s University
Halifax, Nova Scotia, B3H 3C3, Canada
Bert.Hartnell@smu.ca

We say that a graph $G$ is in the collection $M_t$ if there are precisely $t$ different sizes of maximal independent sets of vertices in $G$. Thus the $M_1$ graphs are the well-covered ones (introduced by M. Plummer) where all the maximal independent sets are of one size. Some preliminary observations of the role vertices of degree one (leaves) play in a graph of higher girth belonging to $M_t$ will be outlined.

Computing a Poset Structure for Twisted Involutions in Weyl Groups

Ruth Haas and Aloysius G. Helminck*
Professor Department of Mathematics, North Carolina State University
Raleigh, NC 27695-8205
lock@unity.ncsu.edu

Let $(W,S)$ be a finite Coxeter system, and $\theta$ an involution that fixes a basis for the root system associated with $W$. We show that the set of $\theta$-twisted involutions in $W$, $I_\theta = \{ w \in W : \theta(w) = w^{-1} \}$ is in one to one correspondence with the set $I_{id}$ of regular involutions in $W$. The elements of $I_\theta$ are characterized by sequences in $S$ which induce an ordering called the Bruhat order. In particular, for irreducible root systems, the ascending Bruhat order of $I_\theta$, for nontrivial $\theta$ is identical to the descending Bruhat order of $I_{id}$.

The Number of Repeated Blocks in Twofold Extended Triple Systems

Wen-Chung Huang* and Fu-Chang Ke
Department of Mathematics, Soochow University
Taipei, Taiwan, Republic of China
wchuang@math.scu.edu.tw

A twofold extended triple system with two idempotent elements, $TETS(v)$, is a pair $(V,B)$, where $V$ is a $v$-set and $B$ is a collection of unordered triple, called block, of type $\{x,y,z\}$, $\{x,x,y\}$ or $\{x,x,x\}$ such that each pair (not necessarily distinct) belongs to exactly two triples and there is only two triple of the type $\{x,x,x\}$.
This paper shows that an indecomposable $TETS(v)$ exists which contains exactly $k$ pairs of repeated blocks if and only if $v \not\equiv 0$ mod 3, $v \geq 5$ and $0 \leq k \leq b_v - 2$, where $b_v = (v+2)(v+1)/6$. 
**Group-Magic Cartesian Products**

Dawn M. Jones  
*Department of Mathematics, SUNY Brockport*  
Brockport, NY 14420  
djones@brockport.edu

Let $A$ be an abelian group. We say that a graph is $A$-magic if there is a labeling using $A - \{0\}$ on the set of edges that induces a constant labeling on the set of vertices. In this paper we examine the $A$-magic property of cartesian products of trees and regular graphs.

**New Results in Graph Packing**

Hemanshu Kaul* and Alexandr Kostochka  
*Department of Mathematics, University of Illinois at Urbana-Champaign*  
Urbana, IL 61801  
hkaul@math.uiuc.edu

Let $G_1$ and $G_2$ be graphs of order at most $n$, with maximum degree $\Delta_1$ and $\Delta_2$, respectively. We say that $G_1$ and $G_2$ pack if their vertex sets map injectively into $[n]$ so that the images of the edge sets are disjoint. Note that the concept of graph packing generalizes various extremal graph problems, including problems on forbidden subgraphs, fixed subgraphs and equitable coloring. The study of packings of graphs was started in the 1970s by Sauer and Spencer and by Bollobås and Eldridge.

Sauer and Spencer showed that if $\Delta_1\Delta_2 < \frac{n}{2}$, then $G_1$ and $G_2$ pack. We extend this by characterizing the extremal graphs: if $\Delta_1\Delta_2 \leq \frac{n}{2}$, then $G_1$ and $G_2$ fail to pack if and only if one of $G_1$ or $G_2$ is a perfect matching and the other either is $K_{\frac{n}{2}, \frac{n}{2}}$ with $\frac{n}{2}$ odd or contains $K_{\frac{n}{2} + 1}$. This result can thought of as small step towards the well-known Bollobås-Eldridge graph packing conjecture. We will also mention other new work related to this conjecture.

**Distinguishing Chromatic Number of Cartesian Products of Graphs**

Hemanshu Kaul*, J. Choi, and S. Hartke  
*Department of Mathematics, University of Illinois at Urbana-Champaign*  
Urbana, IL 61801

The distinguishing chromatic number of a graph $G$, $\chi_D(G)$, is the least number of colors needed for a proper coloring of $G$ with the property that the only color-preserving automorphism of $G$ is the identity. That is, we want a proper coloring of a graph that breaks all its symmetries, so that the coloring together with the structure of the graph uniquely determines the vertices. This is an extension of both the chromatic number and the distinguishing number of graphs.

The chromatic number, $\chi(G)$, is an immediate lower bound for $\chi_D(G)$. We show that $\chi_D(G)$ can be surprisingly at most one worse than $\chi(G)$ for $G$ a Cartesian power of any graph. The main theorem is: For every graph $G$, there exists a constant $d_G$ such that for all $d \geq d_G$, $\chi_D(G^d) \leq \chi(G) + 1$, where $G^d$ denotes the Cartesian product of $d$ copies of $G$. Along the way, we also find the distinguishing chromatic number for Hypercubes, Cartesian products of complete graphs (Hamming graphs), and Cartesian products of complete multipartite graphs.
Comparing Subclasses of Well-Covered Graphs

Erika L.C. King
Department of Mathematics and Computer Science, Hobart and William Smith Colleges
Geneva, NY 14456
eking@hus.edu

A graph $G$ is said to be well-covered if every maximal independent set of $G$ is of the same size. It has been shown that characterizing well-covered graphs is a co-NP-complete problem. In an effort to characterize some of these graphs, different subclasses of well-covered graphs have been studied. In this talk, we will discuss the relationships between four of these subclasses: well-dominated graphs (those graphs for which every minimal dominating set is minimum), $\alpha = \gamma$ graphs (those graphs for which the cardinality of a maximum independent set is the same as the cardinality of a minimum dominating set), strongly well-covered graphs (those well-covered graphs that remain well-covered with the deletion of any edge), and stable well-covered graphs (those well-covered graphs - introduced in the speaker’s doctoral dissertation - that remain well-covered with the addition of any edge). We illustrate which of these subclasses intersect, which are subsets of one another and which are disjoint from one another.

$Q(a)P(b)$-Super Edge-Gracefulness of Hypercubes

Harris Kwong* and Sin-Min Lee
Department of Mathematical Sciences, State University of New York at Fredonia
Fredonia, NY 14063
kwong@fredonia.edu

Let $a, b$ be two positive integers. Given a graph $G$ with $|V(G)| = p$ and $|E(G)| = q$, define two sets $Q(a)$ and $P(b)$ as follows:

\[
Q(a) = \begin{cases} 
\{\pm a, \pm (a+1), \ldots, \pm (a+(q-2)/2)\} & \text{if } q \text{ is even,} \\
\{0\} \cup \{\pm a, \pm (a+1), \ldots, \pm (a+(q-3)/2)\} & \text{if } q \text{ is odd.}
\end{cases}
\]

\[
P(b) = \begin{cases} 
\{\pm b, \pm (b+1), \ldots, \pm (b+(p-2)/2)\} & \text{if } p \text{ is even,} \\
\{0\} \cup \{\pm b, \pm (b+1), \ldots, \pm (b+(p-3)/2)\} & \text{if } p \text{ is odd.}
\end{cases}
\]

The graph $G$ is said to be $Q(a)P(b)$-super edge-graceful, in short $Q(a)P(b)$-SEG, if there exist onto mappings $f : E(G) \to Q(a)$ and $f^+ : V(G) \to P(b)$ such that $f^+(v) = \sum_{uv \in E(G)} f(uv)$. We study the values of $a$ and $b$ for which the hypercube $Q_3$ is $Q(a)P(b)$-SEG.
Edge-Connectivity and Edge-Disjoint Spanning Trees

Paul A. Catlin, Hong-Jian Lai*, and Yehong Shao
Department of Mathematics, West Virginia University
Morgantown, WV 26506
hjlai@math.wvu.edu

Given a graph $G$, for an integer $c \in \{2, \cdots, |V(G)|\}$, define
\[
\lambda_c(G) = \min\{|X| : X \subseteq E(G), \omega(G - X) \geq c\}.
\]
For a graph $G$ and for an integer $c = 1, 2, \cdots, |V(G)| - 1$, define,
\[
\tau_c(G) = \min_{X \subseteq E(G) \text{ and } \omega(G - X) > c} \frac{|X|}{\omega(G - X) - c},
\]
where the minimum is taken over all subsets $X$ of $E(G)$ such that $\omega(G - X) - c > 0$. In this paper, we establish a relationship between $\lambda_c(G)$ and $\tau_{c-1}(G)$, which gives a characterization of the edge-connectivity of a graph $G$ in terms of the spanning tree packing number of subgraphs of $G$. The digraph analogue is also obtained. The main results are applied to show that if a graph $G$ is $s$-hamiltonian, then $L(G)$ is also $s$-hamiltonian, and that if a graph $G$ is $s$-hamiltonian-connected, then $L(G)$ is also $s$-hamiltonian-connected.

Graph Theory and Cheeger Constants of Hyperbolic 3-Manifolds

Dominic Lanphier
Department of Mathematics, Western Kentucky University
Bowling Green, KY 42101-1078
Dominic.Lanphier@wkul.edu

We show how estimates of the isoperimetric numbers of certain Cayley graphs can give estimates on the Cheeger constants of certain hyperbolic 3-manifolds.
All Trees of Odd Order with Three Even Vertices are Super Edge-Graceful

Sin-Min Lee* and Yong-Song Ho
Department of Computer Science, San Jose State University
San Jose, California 95192

A \((p, q)\)-graph \(G\) is said to be edge graceful if the edges can be labeled by \(1, 2, \cdots, q\) so that the vertex sums are distinct, mod \(p\). It is shown that if a tree \(T\) is edge-graceful then its order must be odd. Lee conjectured that all trees of odd orders are edge-graceful. Lee showed that every tree of odd order with one even vertex is edge-graceful. A graph \(H = (V, E)\) of order \(p\) and size \(q\) is said to be super edge-graceful if there exists a bijection

\[
    f : E \rightarrow \{0, +1, -1, +2, -2, \cdots, (q-1)/2, -(q-1)/2\} \text{ if } q \text{ is odd},
    f : E \rightarrow \{+1, -1, +2, -2, \cdots, (q-1)/2, -(q-1)/2\} \text{ if } q \text{ is even},
\]

such that the induced vertex labeling \(f^*\) defined by \(f^*(u) = \sum\{f(u, v) : (u, v) \in E\}\) has the property:

\[
    f^* : V \rightarrow \{0, +1, -1, +2, -2, \cdots, (q-1)/2, -(q-1)/2\} \text{ if } q \text{ is odd},
    f^* : V \rightarrow \{+1, -1, +2, -2, \cdots, (q-1)/2, -(q-1)/2\} \text{ if } q \text{ is even}.
\]

is a bijection. Mitchem and Simoson introduced the concept of super edge-graceful graphs which is a stronger concept than edge-graceful for trees. We show that every tree of odd order with three even vertices is super edge-graceful.

On \(Q(a)P(b)\)-Super Edge-Graceful \((p, p+1)\)-Graphs

Sin-Min Lee*, Yung-Chin Wang, and Yihui Wen
Department of Computer Science, San Jose State University
San Jose, California 95192

Let \(a, b\) be two positive integers, for the graph \(G\) with vertex set \(V(G)\) and edge set \(E(G)\) with \(p = |V(G)|\) and \(q = |E(G)|\), we define two sets \(Q(a)\) and \(P(b)\) as follows:

\[
    Q(a) = \begin{cases} 
    \{\pm a, \pm(a + 1), \ldots, \pm(a + (q-2)/2)\} & \text{if } q \text{ is even}, \\
    \{0\} \cup \{\pm a, \pm(a + 1), \ldots, \pm(a + (q-3)/2)\} & \text{if } q \text{ is odd};
    \end{cases}
\]

\[
    P(b) = \begin{cases} 
    \{\pm b, \pm(b + 1), \ldots, \pm(b + (p-2)/2)\} & \text{if } p \text{ is even}, \\
    \{0\} \cup \{\pm b, \pm(b + 1), \ldots, \pm(b + (p-3)/2)\} & \text{if } p \text{ is odd}.
    \end{cases}
\]

For the graph \(G\) with \(p = |V(G)|\) and \(q = |E(G)|\), \(G\) is said to be \(Q(a)P(b)\)-super edge-graceful (in short \(Q(a)P(b)\)-SEG), if there exists a function pair \((f, f^+\)) which assigns integer labels to the vertices and edges; that is, \(f^+ : V(G) \rightarrow P\) and \(f : E(G) \rightarrow Q\) such that \(f^+\) is onto \(P\) and \(f\) is onto \(Q\), and \(f^+(u) = \sum\{f(u, v) : (u, v) \in E(G)\}\).

We investigate \(Q(a)P(b)\) super-edge-graceful \((p, p+1)\)-graphs.
Structural Properties of Hyper-stars

Eddie Cheng, Laszlo Liptak*
Department of Mathematics and Statistics, Oakland University
Rochester, MI 48309
liptak@oakland.edu

Star graphs were introduced as a competitive model of the $n$-cubes, then hyper-star graphs were introduced to be a competitive model to both $n$-cubes and star-graphs. The hyper-star $HS(n, k)$ has vertex set of all $\{0, 1\}$-strings of length $n$ with exactly $k$ 1’s, two vertices being adjacent if and only if one can be obtained from the other by exchanging the first symbol with a different symbol (1 with 0, or 0 with 1) in another position. We discuss strong connectivity properties and orientability of these graphs.

Graph Reconstruction Numbers

Brian McMullen*, and Stanislaw Radziszowski
Rochester Institute of Technology,
Rochester, NY 14623
bmm3056@rit.edu

The Graph Reconstruction Conjecture is one of the most important open problems in graph theory today. Proposed in 1942, the conjecture posits that every simple, finite, undirected graph with three or more vertices can be uniquely reconstructed up to isomorphism given the multiset of subgraphs produced by deleting each vertex of the original graph. Related to the Reconstruction Conjecture, reconstruction numbers concern the minimum number of vertex deleted subgraphs required to uniquely identify a graph up to isomorphism.

In this talk, we discuss the resulting data from calculating reconstruction numbers for all graphs with up to ten vertices. From this data, we establish the reasons behind all large existential reconstruction numbers ($\exists rn(G) > 3$) identified within the set of graphs $G$, where $3 \leq |V(G)| \leq 10$ and identify new classes of graphs that have large reconstruction numbers.

We also consider 2-reconstructibility – the ability to reconstruct a graph $G$ from the multiset of subgraphs produced by deleting each combination of two vertices from $G$. The 2-reconstructibility of all graphs with nine or less vertices was tested, identifying four graphs in this range with five vertices as the highest order of graphs that are not 2-reconstructible.
Necessary and Sufficient Conditions for Tiling with $4 \times 6$ and $5 \times 7$ Rectangles

Rachell Ashley, Aisosa Ayela-Uwangue, Frances Cabrera
Carol Callesano, Darren A. Narayan*, and Allen J. Schwenk

Department of Mathematics & Statistics, Rochester Institute of Technology
Rochester, NY 14623
dansma@rit.edu

Problem B-3 on the 1991 William Lowell Putnam Examination asked "Does there exist a natural number $L$ such that if $m$ and $n$ are integers greater than $L$, then an $m \times n$ rectangle may be expressed as a union of $4 \times 6$ and $5 \times 7$ rectangles, any two intersect at most along their boundaries?" Narayan and Schwenk proved that the minimum value for $L$ is 33. However the case involving rectangles with a dimension less than or equal to 33 is still unsolved.

Our goal is to determine a definitive list of which rectangles can be tiled using $4 \times 6$ and $5 \times 7$ tiles, and which rectangles cannot. We use methods from combinatorics and integer programming to reduce the number of unsolved cases to 18.

On Graphs and Their Irreducible Subgraphs

Masanori Koyama, David L. Neel*, and Michael Orrison

Mathematics Department, Seattle University
Seattle, WA 98122-1090
neel@seattleu.edu

We present an algorithm which associates to each graph $G$ an induced subgraph $I(G)$ which can be used to more easily compute the linear complexity of $G$. If $G = I(G)$, then we call $G$ irreducible. We present structural characteristics of irreducible graphs, along with relationships and similarities between $G$ and $I(G)$.

On Friendly Index Sets of Cycles Root-Union of Stars

Yong-Song Ho, Sin-Min Lee, and Ho Kuen Ng*

Department of Mathematics, San Jose State University
San Jose, CA 95192
ng@math.sjsu.edu

Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$. A labeling $f : V(G) \to \mathbb{Z}_2$ induces an edge labeling $f^* : E(G) \to \mathbb{Z}_2$ defined by $f^*(xy) = f(x) + f(y)$, for each edge $xy \in E(G)$. For $i \in \mathbb{Z}_2$, let $v_f(i) = \text{card}\{v \in V(G) : f(v) = i\}$ and $e_f(i) = \text{card}\{e \in E(G) : f^*(e) = i\}$. A labeling $f$ of a graph $G$ is said to be friendly if $|v_f(0) - v_f(1)| \leq 1$. The friendly index set of the graph $G$, $FI(G)$, is defined as $\{e_f(0) - e_f(1) : \text{ the vertex labeling } f \text{ is friendly}\}$. This is a generalization of graph cordiality. We introduce a graph construction called the root-union and investigate when gaps exist in the friendly index sets of cycles root-union of stars.
The Sequential Sum Problem and the On-line Steiner Problem

Zevi Miller, Dan Pritikin, Manley Perkel*, and I. H. Sudborough
Department of Mathematics and Statistics, Wright State University
Dayton, OH 45435
manley.perkel@wright.edu

Given a connected, edge-weighted graph and a subset $S$ of the vertices, the Steiner problem is to find the minimum weight of any subtree containing $S$, and if possible, construct such a tree (called a Steiner tree for $S$).

Algorithms for doing this naturally require knowledge of the entire set $S$. What if the $n$ vertices of $S$ are revealed to us one vertex at a time in a dynamic or on-line fashion? Having already selected a tree for connecting the first $k$ vertices, when the $(k+1)$st is revealed, we are to expand the tree to include this vertex as well, attempting ultimately to approximate a Steiner tree for $S$. In 1991, Imase and Waxman introduced this on-line Steiner problem. Given an algorithm for selecting such trees on-line, they assigned it a performance ratio, relative to the weights of the Steiner trees for connecting the vertices so far revealed at each stage. They showed a lower bound of $1 + \frac{1}{2} \log_2(n-1)$ for this ratio and made the conjecture that there is some on-line algorithm whose performance ratio achieves this bound.

In this paper we introduce the Sequential Sum Problem and relate it and its measures of performance to that of the on-line Steiner problem. We show that a strong form of the greedy algorithm achieves a performance ratio that converges to the conjectured $1 + \frac{1}{2} \log_2 n + O(1)$ as the proportion of degree 2 vertices in the graph grows. Our results also imply improvements in certain cases on the known upper bound $\log_2 n$ for the performance ratio of the greedy algorithm.

Chip-Firing Games and Cayley’s Formula

David Perkins* and Mark Kayll
Department of Mathematics, Houghton College
Houghton, NY 14744
David.Perkins@houghton.edu

In counting chip configurations arising during a sequence of “chip-firing games” on a graph, we found a new proof of Cayley’s Formula enumerating the spanning trees of a complete graph. The proof of our chip configuration result relies on a proof by double induction, which I’ll make accessible to both undergraduate and graduate students. I’ll also outline the Cayley connection.
**Coloring Algorithms for Placing Guards in Art Galleries**

Val Pinciu  
Department of Mathematics, Southern Connecticut State University  
New Haven, CT 06515  
pinciuv@southernct.edu

We provide coloring arguments to prove several variations of the original Art Gallery Problem. A set \( G \) of points (the guards) in a simple closed polygon \( P_n \) (the art gallery) is a guard set if every point in the polygon is visible to some point in \( G \). A guard set where every guard is visible by at least some other guard is called guarded guard set. A guard set where the visibility graph is connected is called a connected guard set. We provide sharp bounds for the minimum number of guards, guarded guards, and connected guards necessary to cover a polygonal region. We also consider the traditional galleries where each interior angle is 90° or 270°, as well as galleries with holes.

**Combinatorial Analysis and Independent Sets in Simple Graphs**

P. Pongsunpun  
Department of Mathematics and Computer Science, King Mongkut’s Institute of Technology  
Ladkrubang, Bangkok 10520, Thailand  
kppanut@kmitl.ac.th

A graph with no multiple edge or loop is called simple graph. A simple graph \( S = (V, E) \) consists of \( V \), a nonempty set of nodes or vertices and \( E \), a set of unordered pairs of distinct elements of \( V \) called edges. An independent set in a graph is a set of vertices no two of them are joined by an edge. In this paper, we use analysis of combinatorics to find the number of independent sets in simple graphs. We find the relation between combinatorics and independent set. The general formulas for the number of independent sets of size \( c \) (\( c \) is nonnegative number) are accomplished. We prove all formulas that we obtained. Two types of simple graphs such that paths and cycles are presented.

**Some Applications of Spanning Trees in \( K_{s,t} \)**

Austin Mohr and T.D. Porter*  
Department of Mathematics, Southern Illinois University  
Carbondale, IL 62901  
tporter@math.siu.edu

We partition the set of spanning trees contained in the complete graph \( K_n \) into spanning trees contained in the complete bipartite graph \( K_{s,t} \). This classification shows that many properties of spanning trees in \( K_n \) can be derived from trees in \( K_{s,t} \). We enumerate the trees in \( K_n \) and \( K_{s,t} \) recursively, and after applying the principle of inclusion-exclusion, we obtain some combinatorial and numerical identities, including identities involving expressions of the form \( n^p \), where \( n \) and \( p \) are integers. We generate, store, and do isomorphism testing for certain spanning trees, and present some preliminary data.
The Expansion Properties of Block Design Graphs

D. Lanphier, C. Miller, J. Rosenhouse*, and A. Russell

Department of Mathematics, James Madison University
Harrisonburg, VA 22801
rosenhjd@jmu.edu

A bipartite graph can be associated to any block design in the following manner: The vertices of the graph are the points and blocks of the design. A point vertex is connected to a block vertex if the point is contained in the block. In this talk we will derive upper and lower bounds on the Cheeger constants of the graphs obtained in this manner. Particular attention will be paid to the special cases of symmetric and resolvable designs.

Graphs with Two Vertices of the Same $P_3$-Degrees

Ebrahim Salehi

Department of Mathematics, University of Nevada Las Vegas
Las Vegas, NV 89154-4020
salehi@unlv.nevada.edu

For a given graph $F$, the $F$-degree of a vertex $v$ in $G$, denoted by $F - \text{deg} v$, is the number of subgraphs of $G$ isomorphic to $F$ that contain $v$. A graph $G$ is said to be $F$-regular if the $F$-degrees of all the vertices of $G$ are the same and it is called $F$-irregular if the $F$-degrees of the vertices of $G$ are distinct.

In this paper, we will consider $P_3$-degrees and present some results such as; the existence of graphs with distinct $P_3$-degree vertices and graphs that have exactly two vertices of the same $P_3$-degrees.

Finding Points on Elliptic Curves over Characteristic 2 in Deterministic Polynomial Time

Andrew Shallue

Department of Mathematics, University of Wisconsin-Madison
Madison, WI 53706
shallue@math.wisc.edu

While fast probabilistic algorithms for finding points on elliptic curves over finite fields are well-known, the problem of derandomizing these algorithms is not so well studied. For elliptic curves over fields of characteristic two, we give an algorithm for finding a point in deterministic polynomial time. We also extend the algorithm to the encoding problem, so that now messages can be encoded on such curves in deterministic time for use in elliptic curve cryptography, in particular the ECC analog of Massey-Omura.
Holey Knight’s Tours

Darren A. Narayan and Shelley K. Speiss*
Department of Mathematics & Statistics, Rochester Institute of Technology
Rochester, NY 14623
sks1764@rit.edu

A holey knight’s tour is a knight’s tour on an $m \times n$ board where one or more squares have been removed. We consider boards where $m$ and $n$ are odd and the corner squares are colored black. We investigate the presence of holey knight’s tours on boards missing a single black square. In particular, we show the existence of a holey knight’s tour on $m \times n$ boards where $m$ and $n$ are at least 7 and any black square is removed. Finally, we explore the general problem of removing more than one square.

The Ramsey Numbers $R(C_i, C_j, C_k)$

Kung-Kuen Tse
Department of Mathematics and Computer Science, Kean University
Union, NJ 07083, USA
ktse@kean.edu

The Ramsey number $R(C_i, C_j, C_k)$ is the smallest positive integer $m$ such that any edge coloring with three colors of the complete graph $K_m$ must contain a monochromatic cycle $C_s$, $s = i, j$ or $k$. In this work, we compute the Ramsey numbers $R(C_3, C_4, C_5)$, $R(C_4, C_4, C_5)$ and $R(C_4, C_5, C_5)$. We also verify the known Ramsey numbers $R(C_3, C_3, C_3)$, $R(C_3, C_4, C_4)$ and $R(C_3, C_4, C_4)$ by enumerating all the critical graphs of the relevant order. The results are based on computer algorithms.

Nearly Orthogonal Latin squares

C. Pak Li and G.H.J. Van Rees*
Department of Computer Science, University of Manitoba
Winnipeg, Manitoba Canada R3T 2N2
vanrees@cs.umanitoba.ca

A Latin square of order $n$ is an $n$ by $n$ array in which every row and column is a permutation of a set $N$ of $n$ elements. Let $L = [l_{i,j}]$ and $M = [m_{i,j}]$ be two Latin squares of order $n$, $n$ even, based on the same $N$-set. Define the superposition of $L$ onto $M$ to be the $n$ by $n$ array $A = [l_{i,j}, m_{i,j}]$. Then $L$ and $M$, are said to be nearly orthogonal if the superposition of $L$ onto $M$ has every ordered pair $(i, j)$ appearing exactly once except when $i = j$ when the ordered pair appears 0 times and except for $i - j \sim n/2 (modn)$ when the ordered pair appears 2 times. A set of $t$ Latin squares of order $2m$ is called a set of mutually nearly orthogonal Latin squares ($t \text{NMOLS}(2m)$ if every pair of Latin squares are nearly orthogonal. We give two elementary proofs for theorems proved previously with heavier mathematics. We also give some computer results and conjectures.
Proof of the Erdos-Faudree Conjecture on Quadrilaterals

Hong Wang
Department of Mathematics, University of Idaho
Moscow, Idaho 83844-1103
hwang@uidaho.edu

If $G$ is a graph of order $4k$ and the minimum degree of $G$ is at least $2k$, then $G$ contains $k$ disjoint cycles of length 4.

The Half-Life of Chocolate: a bit of Combinatorial Chemistry

Paul Wilson
Department of Mathematics & Statistics, Rochester Institute of Technology
Rochester, NY 14623
prousma@rit.edu

Consider a set of 100 candy bars numbered 1, 2, ..., 100. Each day a number is drawn at random, with replacement, from the set 1,2,...,100 and the bar with that number, if it isn’t already gone, is eaten. What is the expected number of days until 50 bars have been eaten? 87? The answer involves Stirling numbers, partitions, falling factorials, and a surprise non-combinatorial solution. The problem is worked for samples from 1,2,...,N, but I start with the explicit example N=100.

The Wiener Indices and Polynomials of Graphs Induced by Subdivision Operators

Weigen Yan, Bo-Yin Yang*, and Yeong-Nan Yeh
Department of Mathematics, Tamkang University
Tamsui, Taiwan
by@muscito.org

The sum of distances between all vertices pairs in a connected graph is known as the Wiener Index. It is the earliest of the indices that correlates well with many physicochemical properties of organic compounds and as such has been well-studied over the last quarter of a century. A q-analogue of this Index, termed the Wiener Polynomial by Hosoya but also known today as the Hosoya Polynomial, extends this concept by trying to capture the complete distribution of distances in the graph. The mathematicians have studied several operators on a connected graph in which we see a subdivision of the edges. Herein we show how the Wiener Index of a graph changes with these operations, and the extend the results to Wiener Polynomials.
On a Graph Packing Conjecture
by Bollobas and Eldridge

Hemanshu Kaul, Alexandr Kostochka, and Gexin Yu*
Department of Mathematics, University of Illinois
Urbana, IL 61801
gexinyu@uiuc.edu

Graph packing is one of the basic notions in graph theory. Two graphs $G_1$ and $G_2$ pack if $G_1$ and $G_2$ can be embedded into the same vertex set such that the image of edge sets do not intersect. Bollobás and Eldridge conjectured that if $|V(G_1)| = |V(G_2)| = n$ and $(\Delta(G_1) + 1)(\Delta(G_2) + 1) \leq n + 1$, then $G_1$ and $G_2$ pack. This is one of main conjecture in this area. We study the following weakening of this conjecture: if $|V(G_1)| = |V(G_2)| = n$ and $(\Delta(G_1) + 1)(\Delta(G_2) + 1) \leq 0.5n(1+\epsilon) + 1$ with $0 \leq \epsilon \leq 1$, Do $G_1$ and $G_2$ pack? We prove that for $\epsilon \leq 0.1956$, $G_1$ and $G_2$ pack.

Isolated Toughness and Existence of $[a,b]$-factors in Graphs

Yinghong Ma and Qinglin R. Yu*
Department of Mathematics and Statistics, Thompson Rivers University
Kamloops, BC Canada, V2C 5N3
yu@tru.ca

Let $i(G)$ be the number of isolated vertices in a graph $G$, the isolated toughness of $G$ is defined as $I(G) = \min\{|S|/i(G - S) \mid S \subseteq V(G), i(G - S) \geq 2\}$, if $G$ is not complete; $I(G) = |V(G)| - 1$, otherwise. Let $a$, $b$ be positive integers with $2 \leq a < b$. We proved that if $G$ is a graph with $\delta(G) \geq a$ and $I(G) \geq a$, then $G$ has a fractional $a$-factor. Moreover, if $\delta(G) \geq a$, $I(G) > (a - 1) + \frac{a-1}{b}$ and $G - S$ has no $(a - 1)$-regular component for any subset $S$ of $V(G)$, then $G$ has an $[a,b]$-factor. Furthermore, the bounds in these results are sharp.

Direct Quadrilaterals in a Direct Graph

DanHong Zhang* and Hong Wang
Department of Mathematics, Utica college
Utica, NY 13502-4892
dzhang@utica.edu

Let $D$ be a directed graph of order $4k$, where $k$ is a positive integer. Suppose that the minimum degree of $D$ is at least $6k - 2$. We show that $D$ contains $k$ disjoint directed quadrilaterals with only one exception.