Playing With Mazes

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1. What is a Maze?

A maze or labyrinth is a network of passages, usually intricate and confusing. What counts as intricate and confusing depends, of course, on the mind of the beholder. For most of this paper I shall be concerned with artificial beholders, namely, computers, and how algorithms and data structures can be specified for the automated construction and solving of mazes. For this reason, the mazes to be studied need not be particularly complex by human standards; I shall be investigating generalized principles which will apply to any degree of complexity of mazes — or, rather, of certain subclasses of mazes. Most of the presentation will be non-formal, as befits the subject matter.

The intersection of two passages will be called a **cell** (or **room**) of a maze such that there is a pathway from one passage to the other via that intersection. A maze is **planar** if it is constructable on a (two dimensional) surface such that passages do not cross except at such intersections. Nonplanar mazes would be three (or higher) dimensional networks of passages and rooms.

The principles of maze construction and solution do not change for higher dimensional mazes, and so, for the sake of clarity and ease of drawing diagrams, I shall confine my attention to planar mazes only.

A passage may have a "cost" associated with it, perhaps in terms of the length of the passage, or in terms of hazards, or in terms of tiny trolls who threaten the traveler with unhappy consequences if some suitable tribute is not paid. Or sometimes even if it is paid. A system of roads in a city might be representable as a maze (though not necessarily planar). So might the playing field

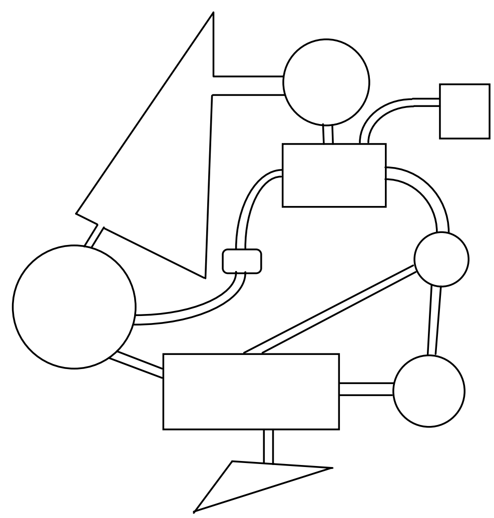

Figure 1. A fictitious maze in the courtyard of Mon, the fictitious emperor of the fictitious realm of D-D, described in a fictitious science fiction novel. The big circle is the peanut gallery.

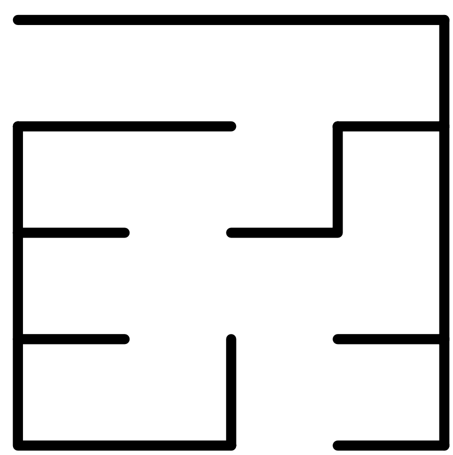

Figure 2. The floor plan of my grandparents' house, roughly as I conceived it as a child. (It bears little resemblance to the actual floor plan.)

of a pinball machine, the stacks in a library, a bureaucracy, a judicial system, a flow chart for a computer program, a crosswords puzzle, or the circulatory system of a door mouse.

The pictorial presentation of a maze might take different forms: the cells and passages might be explicitly drawn (figure 1); the passages might be of length zero, so that adjacent cells have **doors** and **walls** (or open and closed doors) (figure 2); or the cells might be of zero size, existing only conceptually at the intersections of passages (figure 3); finally, any maze may be represented abstractly as a **graph**, with nodes (vertices) and arcs (edges) (figure 4).

I shall be concerned in this paper only with mazes of the second type, which I shall call **regular floor plan (RFP) mazes**, and which are a subset of mazes representable as graphs. Somehow, RFP mazes seem to me to be "traditional" in a way the others are not; but this may be only a childhood prejudice. In any case, it will simplify some of the discussions; they represent a class of mazes of potentially great complexity; and they are easy to present on a computer screen or printer.

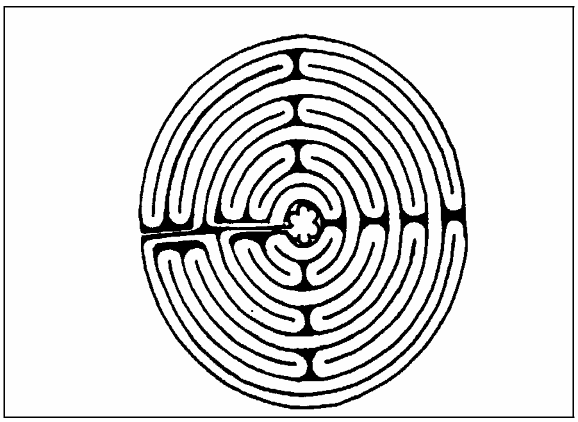

Figure 3. A real maze. A 13th century labyrinth at Chartres cathredral.

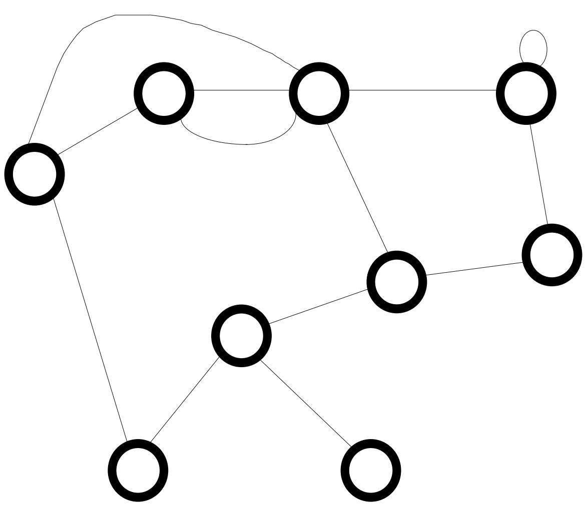

Figure 4. A real abstract maze.

The limitations of RFP mazes are obvious once they are compared with graphs. For example, from one of the nodes in figure 4 there is a passage leading back to the same node. This is not explicitly representable in RFP mazes (except by convention: a maze traveler may move from his present cell immediately to his present cell simply by staying put). Second, graphs may have any number of edges (passages) leading out of or into a node, whereas RFP mazes have no more than four passages (i.e., four immediately accessible neighbors). Third, a graph may be **directed**, such that the passages are one-way only. (RFP mazes might, correspondingly, have "one-way doors" between cells.) Since any maze, including RFP mazes, may be represented as graphs, various theorems from graph theory may be applied to the construction and solution of RFP mazes.

RFP mazes may be either **closed** or **open**. An open maze has an entrance from the outside and/or an exit to the outside, as shown in figure 5. In a closed maze, however, one of the cells might be designated the **start** cell, and one (or more) of the cells might be designated the **goal** cell, as in figure 6. Any open maze may be enlarged so as to become a closed maze which now includes the "entrance" cell and the "exit" cell (figure 7). Closed mazes being the more general variety, our attention will be focused primarily on closed mazes.

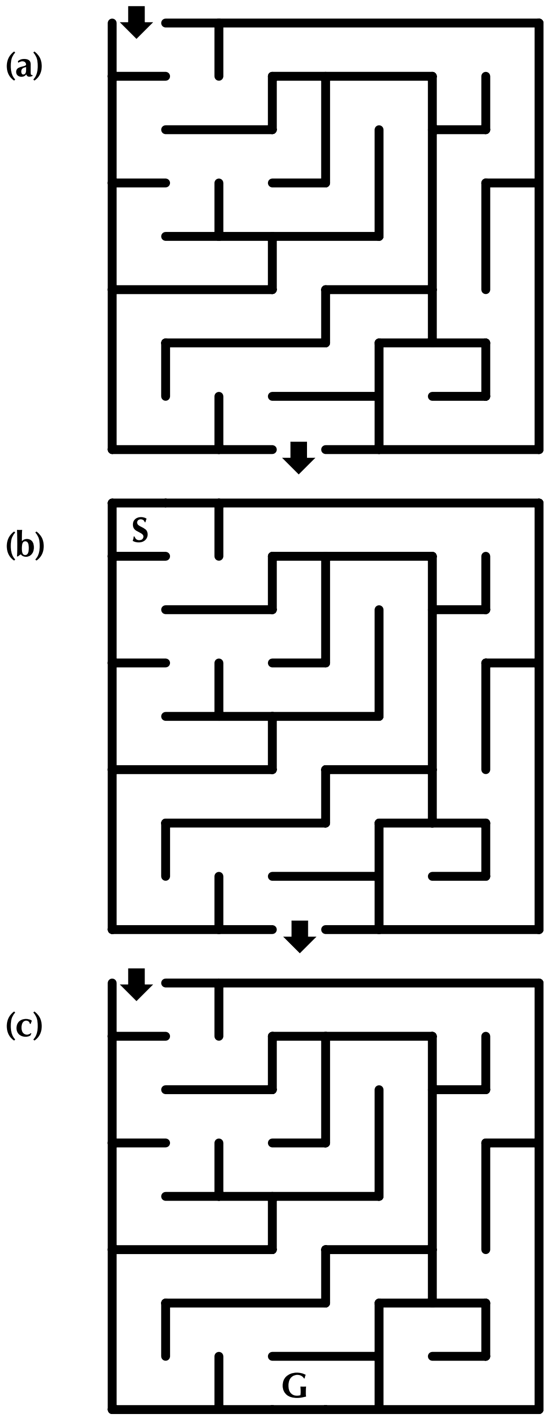

Figure 5. An open maze. (a) Find a path from the entrance to the exit. (b) Find a path from the start cell (inside the maze) to the exit. (I.e., escape from the maze.) (c) Find a path from the entrance to the goal cell (inside the maze). I.e., find the treasure (and possibly find your way back out).

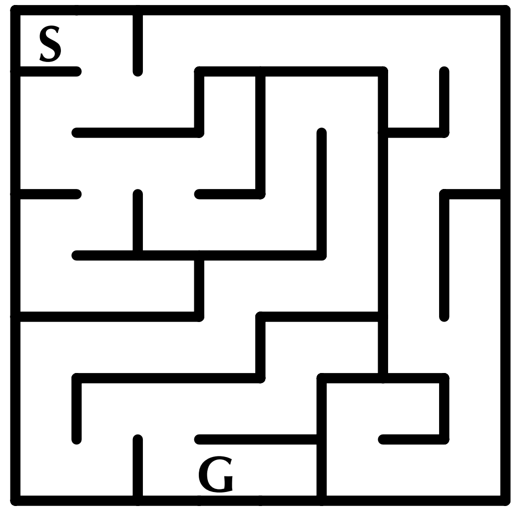

Figure 6. A closed RFP maze. Find a path from the interior start cell, S, to the interior goal cell, G.

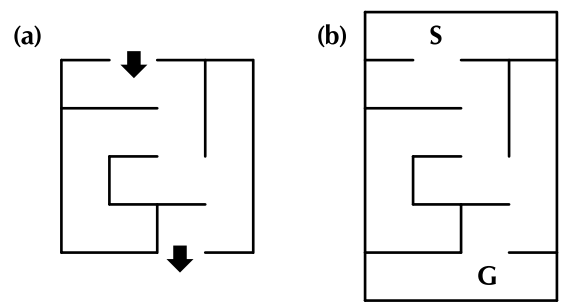

Figure 7. (a) An open maze. (b) A closed maze constructed by enlarging the open maze in (a).