Pulley and Mass Problems

A standard problem for illustrating rotational dynamics is a massive pulley attached by a sting to a linear mass. Let’s look at some of the subtleties involved here.

Q1) Consider the basic pulley and mass setup. We can use the three relations $s = R\theta$, $v=R\omega$, $a = R\alpha$ to relate the angular variables of the wheel to the linear variables of the mass. Note that these merely relate the magnitudes of these quantities and they are not vector equations, because the mass is generally not moving along the direction of the axis used to define the rotational quantities. If we use these three relations in any problem, we must look at our coordinate system and determine if we must insert ± signs into the relations.

Draw a pulley with a mass hanging off the left side and another pulley with the mass hanging off the right side. Write down our usual coordinate system and then list the three equations relating the linear and angular variables in both cases. The equations don’t ‘know’ what side of the wheel we put the string on, so we have to ‘tell’ them this by inserting the correct signs.
Q2) A wheel is shown with a hub of radius 2.0 cm and an outer radius of 5.0 cm. String wrapped around the hub is connected to mass \( m_1 \) and string wrapped around the wheel is attached to mass \( m_2 \). Initially mass \( m_1 \) is moving downwards at 15 cm/s, but it is momentarily at rest after moving 25 cm down.

First, lay down a coordinate system next to the diagram.

a) What is the acceleration of mass \( m_1 \)?

b) What is the wheel’s initial angular velocity, including direction?

c) What is the wheel’s angular acceleration, including direction?

d) What is the initial velocity of mass \( m_2 \), including direction?

e) Through what angle does the wheel rotate before momentarily stopping?

f) How far does \( m_2 \), the striped mass, rise before momentarily stopping?

g) What is the initial radial (centripetal) acceleration of a point at the outer edge of the entire wheel?
Q3) Just as in Physics I, one must often first decide what principle or approach to use in order to solve a given problem. In the case of the pulley problems, it’s usually a toss-up between Newton’s Second Law and Conservation of Energy. In each of the cases below, state which one you would use and why (or why you wouldn’t try to use the other one).

a) You’re asked for the acceleration of the mass?

b) You’re asked for the angular acceleration of the pulley?

c) You’re asked for the velocity of the mass?

d) You’re asked for the angular velocity of the pulley?

e) You’re given a problem with a brake slowing down the pulley (constant tangential frictional force of the rim of the pulley) as the mass falls.

f) You’re given a problem with a motor applying a constant torque to the pulley and winding the mass upwards.
Q4) A pulley is made from a solid disk of radius 15 cm and mass 400 grams, and rotates with no friction. A string passes over the pulley and is attached to masses of $m_1 = 200$ grams and $m_2 = 300$ grams. The system is released from rest.

a) Find the acceleration of $m_2$.
b) Find the tension in each part of the string.
c) Find the speed of $m_2$ after it falls $s = 25$ cm

Note: Since the pulley mass $\neq 0$, it is no longer ideal and the tensions will be different.