Microelectromechanical Systems (MEMs) Electrical Fundamentals

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OUTLINE

- Resistors
- Heaters
- Temperature Sensors
- Capacitors
- Electrostatic Force
- Gap Comb Drives
- Magnetic Actuators
- Piezoelectric Devices
- Diode Temperature Sensors
- Photodiode
- Seeback Effect
- Peltier Effect
- Tunnel Sensors
INTRODUCTION

Resistors are used as heaters, temperature sensors, piezoresistor sensors and photoconductors. Heaters are used in many MEMS applications including ink jet print heads, actuators, bio-mems, chemical detectors and gas flow sensors. Diodes, Capacitors, electrostatic comb drives, magnetic devices, and many other devices are used in many applications. This module will discuss these devices as they apply to MEMS.
SURFACE MICROMACHINED GAS FLOW SENSOR

Upstream Polysilicon Resistor

Polysilicon heater

Downstream Polysilicon Resistor
SURFACE MICROMACHINED GAS FLOW SENSOR

L of heater & resistor = 1mm
W (heater) = 50um
W (resistors) = 20um
Gap = 10um
V applied = 27V to 30.5V
Temp ~600 °C at 26 volts
Lifetime > 10 min at 27 volts (possibly longer, did not test)

Vee Chee Hwang, 2004
BULK MICROMACHINED GAS FLOW SENSOR

Measured Resistance, $V/I=1.2\text{Kohms}$
Theoretical Resistance, $L \times \rho / W = 400\mu\text{m} \times 60/20\mu\text{m} = 1.2\text{Kohms}$

Raunak Mann, 2004
THERMIONIC GAS DETECTOR

- Polysilicon Micro-filament heater

- Make hot
- Thermionic emission occurs causing ionization
- Force ions to a collection plate
- Measure resulting current or voltage

Robert Manley, 2004
THERMIONIC GAS DETECTOR

Polysilicon

Silicon Nitride

Un-etched Sacox

Si
THERMIONIC GAS DETECTOR

Cold

Hot

Robert Manley, 2004
**Resistor** a two terminal device that exhibits a linear I-V characteristic that goes through the origin. The inverse slope is the value of the resistance.

\[ R = \frac{V}{I} = \frac{1}{\text{slope}} \]
Resistance = \( R = \rho \frac{L}{\text{Area}} = \rho_s \frac{L}{w} \) ohms

Resistivity = \( \rho = \frac{1}{q \mu_n n + q \mu_p p} \) ohm-cm

Sheet Resistance = \( \rho_s = 1/ \left( \int q \mu(N) N(x) \, dx \right) \approx 1/(q \mu \text{Dose}) \) ohms/square

\[ \rho_s = \frac{\rho}{t} \]

Rho is the bulk resistivity of the material (ohm-cm)
Rhos is the sheet resistance (ohm/sq) = \( \frac{\rho_s}{t} \)

Note: sheet resistance is convenient to use when the resistors are made of thin sheet of material, like in integrated circuits.

\[ R = \rho_s \frac{L}{\text{Area}} = \rho_s \frac{L}{W} \]
Electron and hole mobilities in silicon at 300 K as functions of the total dopant concentration (N). The values plotted are the results of the curve fitting measurements from several sources. The mobility curves can be generated using the equation below with the parameters shown:

\[
\mu(N) = \mu_{m_i} + \frac{(\mu_{\text{max}} - \mu_{\text{min}})}{\{1 + (N/N_{\text{ref}})^\alpha}\}}
\]

From Muller and Kamins, 3\textsuperscript{rd} Ed., pg 33
TEMPERATURE EFFECTS ON MOBILITY

Derived empirically for silicon for T in K between 250 and 500 °K and for N (total dopant concentration) up to 1 E20 cm-3

\[ \mu_n (T,N) = 88 \ Tn^{-0.57} + \frac{1250 \ Tn^{-2.33}}{1 + \left[ \frac{N}{(1.26E17 \ Tn^{2.4})} \right]^{0.88} \ Tn^{-0.146}} \]

\[ \mu_p (T,N) = 54.3 \ Tn^{-0.57} + \frac{407 \ Tn^{-2.33}}{1 + \left[ \frac{N}{(2.35E17 \ Tn^{2.4})} \right]^{0.88} \ Tn^{-0.146}} \]

Where \( Tn = T/300 \)

From Muller and Kamins, 3rd Ed., pg 33
**EXCELL WORKSHEET TO CALCULATE MOBILITY**

**MICROELECTRONIC ENGINEERING**  
3/13/2005

**CALCULATION OF MOBILITY**  
Dr. Lynn Fuller

To use this spreadsheet change the values in the white boxes. The rest of the sheet is protected and should not be changed unless you are sure of the consequences. The calculated results are shown in the purple boxes.

<table>
<thead>
<tr>
<th>CONSTANTS</th>
<th>VARIABLES</th>
<th>CHOICES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tn = T/300 = 1.22</td>
<td>Temp = 365 °K</td>
<td>1=yes, 0=no</td>
</tr>
<tr>
<td>N total = 1.00E+18 cm⁻³</td>
<td>n-type</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>p-type</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>&lt;100&gt;</td>
<td></td>
</tr>
</tbody>
</table>

Kamins, Muller and Chan; 3rd Ed., 2003, pg 33

mobility = **163** cm²/(V·sec)
EXCELL WORKSHEET TO CALCULATE RESISTANCE

MICROELECTRONIC ENGINEERING
Rochester Institute of Technology
Dr. Lynn Fuller

7/23/2007

CALCULATION OF RESISTANCE FROM LENGTH, WIDTH, THICKNESS AND IMPLANT DOSE

To use this spreadsheet change the values in the white boxes. The rest of the sheet is protected and should not be changed unless you are sure of the consequences. The calculated results are shown in the purple boxes.

Calculation of Mobility

<table>
<thead>
<tr>
<th>CONSTANTS</th>
<th>VARIABLES</th>
<th>CHOICES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_n = T/300 \cdot 1.00$</td>
<td>Temp= $300^\circ$K</td>
<td>1=yes, 0=no</td>
</tr>
<tr>
<td>$N_{\text{total}} = 2.00E+16\text{cm}^{-3}$</td>
<td>n-type</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>p-type</td>
<td>1</td>
</tr>
</tbody>
</table>

Kamins, Muller and Chan; 3rd Ed., 2003, pg 33

$mobility = \mu = 433 \text{cm}^2/(V\text{-sec})$

Calculation of Resistance

$R = \frac{\rho L}{W \cdot t}$

$R = \frac{\rho_{\text{sh}} L}{W}$

$\rho = \text{bulk resistivity}$

$\rho_{\text{sh}} = \text{sheet resistance} = \frac{1}{q \cdot \mu \cdot \text{Dose}}$

$q = 1.6e-19 \text{coul/\text{ion}}$

$Length, L =$ 6000 $\mu$m

$Width, W =$ 20 $\mu$m

$Thickness, t =$ 0.5 $\mu$m

$\text{Implant Dose} = 1.00E+12 \text{ions/cm}^2$

$N_{\text{total}} =$ Average Doping = 2.00E+16 atoms/cm$^3$

$Mobility, \mu =$ 433 cm$^2$/V-sec

$\text{Resistance} =$ 1.73E+09 ohms
TEMPERATURE COEFFICIENT OF RESISTANCE

\[ \Delta R/\Delta T \text{ for semiconductor resistors} \]

\[ R = R_{\text{hos}} \frac{L}{W} = R_{\text{ho}}/t \frac{L}{W} \]

assume \( W, L, t \) do not change with \( T \)

\[ R_{\text{ho}} = \frac{1}{(q\mu n + q\mu p)} \text{ where } \mu \text{ is the mobility which is a function of temperature, } n \text{ and } p \text{ are the carrier concentrations which can be a function of temperature (in lightly doped semiconductors)} \]

as \( T \) increases, \( \mu \) decreases, \( n \) or \( p \) may increase and the result is that \( R \) usually increases unless the decrease in \( \mu \) is cancelled by the increase in \( n \) or \( p \)
RESISTANCE AS A FUNCTION OF TEMPERATURE

\[ R(T,N) = \frac{1}{q \mu_n(T,N) n + q \mu_p(T,N) p} \frac{L}{Wt} \]

L, W, t, are physical length width and thickness and do not change with T
Constant \( q = 1.6 \times 10^{-19} \) coul
\( \mu_n(T,N) \) = see expression on previous page
\( \mu_p(T,N) \) = see expression on previous page
n = Nd and p = 0 if doped n-type
n=0 and p = Na if doped p-type
n = p = ni(T) if undoped, see exact calculation on next page

\[ ni(T) = AT^{3/2} e^{-(E_g)/2KT} = 1.45 \times 10^{10} @ 300 \, ^\circ K \]
**EXACT CALCULATION OF n AND p**

<table>
<thead>
<tr>
<th>CONSTANTS</th>
<th>VARIABLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>1.38E-23 J/K</td>
</tr>
<tr>
<td>q</td>
<td>1.60E-19 Coul</td>
</tr>
<tr>
<td>Ego</td>
<td>1.16 eV</td>
</tr>
<tr>
<td>a</td>
<td>7.02E-04</td>
</tr>
<tr>
<td>B</td>
<td>1.11E+03</td>
</tr>
<tr>
<td>h</td>
<td>6.63E-34 Jsec</td>
</tr>
<tr>
<td>Ego</td>
<td>1.16 eV</td>
</tr>
<tr>
<td>a</td>
<td>7.02E-04</td>
</tr>
<tr>
<td>B</td>
<td>1.11E+03</td>
</tr>
<tr>
<td>n</td>
<td>3.00E+16 cm-3</td>
</tr>
<tr>
<td>p</td>
<td>2.02E+15</td>
</tr>
</tbody>
</table>

Donor Concentration

Acceptor Concentration

Eg = Ego - (aT^2/(T+B))

Nc = 2.82E+19 cm-3

Nv = 1.34E+01 cm-3

Fermi Level, Ef

free electrons, n = Nc exp(-q(Ec-Ef)/KT)

ionized donors, Nd+ = Nd*(1+2*exp(q(Ef-Ed)/KT))^{-1}

holes, p = Nv exp((Ef-Ev)/KT)

ionized acceptors, Na- = Na*(1+2*exp(q(Ea-Ef)/KT))^{-1}

Charge Balance = p + Nd+ - n - Na-
The n-type wafer is always biased positive with respect to the p-type diffused region. This ensures that the pn junction that is formed is in reverse bias, and there is no current leaking to the substrate. Current will flow through the diffused resistor from one contact to the other. The I-V characteristic follows Ohm’s Law: \( I = \frac{V}{R} \)

Sheet Resistance \( = \rho_s \sim \frac{1}{(\text{qµ Dose})} \) ohms/square
Desired resistor network

500 Ω

400

250 Ω

Layout if Rhos = 100 ohms/square
Estimate the sheet resistance of the 4000 ohm resistor shown.
DIFFUSION FROM A CONSTANT SOURCE

\[ N(x,t) = N_0 \text{erfc} \left( \frac{x}{2 \sqrt{D_p t_p}} \right) \]

**Solid Solubility Limit, No**

**Wafer Background Concentration, NBC**

**erfc function**

**p-type**

**n-type**

\[ X_j \]

into wafer
DIFFUSION FROM A LIMITED SOURCE

\[ N(x,t) = Q'_A(tp) \exp\left(-\frac{x^2}{4Dt}\right) \]

where \( D \) is the diffusion constant at the drive in temperature and \( t \) is the drive in diffusion time, \( D_p \) is the diffusion constant at the predeposition temperature, and \( t_p \) is the predeposition time.

For erfc predeposition:

\[ Q'_A(tp) = \frac{Q_A(tp)}{\text{Area}} = 2N_0\sqrt{\frac{Dtp}{\pi}} = \text{Dose} \]
## DIFFUSION CONSTANTS AND SOLID SOLUBILITY

### DIFFUSION CONSTANTS

<table>
<thead>
<tr>
<th>TEMP</th>
<th>BORON PRE or DRIVE-IN Dp or D</th>
<th>PHOSPHOROUS PRE Dp</th>
<th>DRIVE-IN D</th>
<th>BORON SOLID NOB</th>
<th>PHOSPHOROUS SOLID NOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>900 °C</td>
<td>1.07E-15 cm²/s</td>
<td>2.09E-14 cm²/s</td>
<td>7.49E-16 cm²/s</td>
<td>4.75E20 cm⁻³</td>
<td>6.75E20 cm⁻³</td>
</tr>
<tr>
<td>950</td>
<td>4.32E-15</td>
<td>6.11E-14</td>
<td>3.29E-15</td>
<td>4.65E20</td>
<td>7.97E20</td>
</tr>
<tr>
<td>1000</td>
<td>1.57E-14</td>
<td>1.65E-13</td>
<td>1.28E-14</td>
<td>4.825E20</td>
<td>9.200E20</td>
</tr>
<tr>
<td>1050</td>
<td>5.15E-14</td>
<td>4.11E-13</td>
<td>4.52E-14</td>
<td>5.000E20</td>
<td>1.043E21</td>
</tr>
<tr>
<td>1100</td>
<td>1.55E-13</td>
<td>9.61E-13</td>
<td>1.46E-13</td>
<td>5.175E20</td>
<td>1.165E21</td>
</tr>
<tr>
<td>1150</td>
<td>4.34E-13</td>
<td>2.12E-12</td>
<td>4.31E-13</td>
<td>5.350E20</td>
<td>1.288E21</td>
</tr>
<tr>
<td>1200</td>
<td>1.13E-12</td>
<td>4.42E-12</td>
<td>1.19E-12</td>
<td>5.525E20</td>
<td>1.410E21</td>
</tr>
<tr>
<td>1250</td>
<td>2.76E-12</td>
<td>8.78E-12</td>
<td>3.65E-12</td>
<td>5.700E20</td>
<td>1.533E21</td>
</tr>
</tbody>
</table>
ION IMPLANTED RESISTOR

Like the diffused resistor but more accurate control over the sheet resistance. The dose is a machine parameter that is set by the user.

Sheet Resistance = $\rho_s \sim 1/(q\mu \text{ Dose})$  ohms/square

Also the dose can be lower than in a diffused resistor resulting in higher sheet resistance than possible with the diffused resistor.
For polysilicon thin films the Dose = film thickness ,t, x Solid Solubility No if doped by diffusion, or Dose = ion implanter dose setting if implanted

The Sheet Resistance $R_{\text{hos}} = \sim \frac{1}{(\mu \text{ Dose})}$ ohms/square

For metal the Sheet Resistance is $\sim$ the given (table value) of bulk resistivity, $\rho$, divided by the film thickness ,t.

$R_{\text{hos}} = \frac{\rho}{t}$ ohms/square
Final steady state temperature depends on power density in watts/cm²

and

the thermal resistance from heater to ambient

\[ P = IV = I^2R \]  watts
THERMAL CONDUCTIVITY

Rth = 1/C * L/Area

where

C = thermal conductivity
L = thickness of layer between heater and ambient
Area = cross sectional area of the path to ambient
### THERMAL PROPERTIES OF SOME MATERIALS

<table>
<thead>
<tr>
<th>Material</th>
<th>MP °C</th>
<th>Coefficient of Thermal Expansion ppm/°C</th>
<th>Thermal Conductivity w/cmK</th>
<th>Specific Heat cal/gm°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diamond</td>
<td>1412</td>
<td>1.0</td>
<td>20</td>
<td>0.215</td>
</tr>
<tr>
<td>Single Crystal Silicon</td>
<td>1412</td>
<td>2.33</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Poly Silicon</td>
<td>1412</td>
<td>2.33</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Silicon Dioxide</td>
<td>1700</td>
<td>0.55</td>
<td>0.014</td>
<td></td>
</tr>
<tr>
<td>Silicon Nitride</td>
<td>1900</td>
<td>0.8</td>
<td>0.185</td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td>660</td>
<td>22</td>
<td>2.36</td>
<td>0.215</td>
</tr>
<tr>
<td>Nickel</td>
<td>1453</td>
<td>13.5</td>
<td>0.90</td>
<td>0.107</td>
</tr>
<tr>
<td>Chrome</td>
<td>1890</td>
<td>5.1</td>
<td>0.90</td>
<td>0.03</td>
</tr>
<tr>
<td>Copper</td>
<td>1357</td>
<td>16.1</td>
<td>3.98</td>
<td>0.092</td>
</tr>
<tr>
<td>Gold</td>
<td>1062</td>
<td>14.2</td>
<td>3.19</td>
<td></td>
</tr>
<tr>
<td>Tungsten</td>
<td>3370</td>
<td>4.5</td>
<td>1.78</td>
<td></td>
</tr>
<tr>
<td>Titanium</td>
<td>1660</td>
<td>8.9</td>
<td>0.17</td>
<td></td>
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<tr>
<td>Tantalum</td>
<td>2996</td>
<td>6.5</td>
<td>0.54</td>
<td></td>
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<tr>
<td>Air</td>
<td>0</td>
<td>0.00026</td>
<td>0.24</td>
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<tr>
<td>Water</td>
<td>0</td>
<td>0.0061</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

1 watt = 0.239 cal/sec
**HEATER EXAMPLE**

**Example:** Poly heater 100x100µm has sheet resistance of 25 ohms/sq and 9 volts is applied. What temperature will it reach if built on 1 µm thick oxide?

\[
\text{Power} = \frac{V^2}{R} = \frac{81}{25} = 3.24 \text{ watt}
\]

\[
R_{\text{thermal}} = \frac{1}{C} \frac{L}{\text{Area}} = \frac{1}{0.014 \text{ watt/cm } ^\circ \text{C}} (1 \times 10^{-4} \text{ cm}/(100 \times 10^{-4} \text{ cm} \times 100 \times 10^{-4} \text{ cm}))
\]

\[
= 71.4 ^\circ \text{C/watt}
\]

\[
\text{Temperature} = T_{\text{ambient}} + (3.24) (71.4) = T_{\text{ambient}} + 231 ^\circ \text{C}
\]
TEMPERATURE SENSOR EXAMPLE

Example: A diffused heater is used to heat a sample. The temperature is measured with a poly silicon resistor. For the dimensions given what will the resistance be at 90°C and 65°C

\[
R(T,N) = \frac{1}{q \mu_n (T,N) n} \frac{L}{Wt}
\]

\[
t = 1 \mu m \\
Nd = n = 1 \times 10^{18} cm^{-3}
\]

\[
q \mu_n (T=390,N=1e18) n = 1.6e-19 \times 10^{18} = 22.6 \\
R = 2212 \text{ ohms}
\]

\[
q \mu_n (T=365,N=1e18) n = 1.6e-19 \times 163 \times 10^{18} = 26.1 \\
R = 1916 \text{ ohms}
\]
**THERMAL FLOW SENSORS**

Spring 2003
EMCR 890 Class Project
Dr. Lynn Fuller

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**Responsibility:**

Spring 2003
EMCR 890 Class Project
Dr. Lynn Fuller

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**Rochester Institute of Technology**
**Microelectronic Engineering**

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**Page 33**
GAS FLOW SENSORS

Constant heat (power in watts) input and two temperature measurement devices, one upstream, one downstream. At zero flow both sensors will be at the same temperature. Flow will cause the upstream sensor to be at a lower temperature than the downstream sensor.
FLOW SENSOR ELECTRONICS

Constant Power Circuit

R1
Vout
R2
Gnd

+6 Volts

-6 Volts

Vout near Zero so that it can be amplified

AD534

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A single heater/sensor element is placed in the flow. The amount of power supplied to keep the temperature constant is proportional to flow. At zero flow a given amount of power $P_o$ will heat the resistor to temperature $T_o$. With non zero flow more power $P_f$ is needed to keep the resistor at $T_o$. 

**Heater/Sensor**

---

**Flow**
CONSTANT TEMPERATURE CIRCUIT

Analog Divider Using AD534

\[ R = \frac{V}{I} \]

Setpoint

+1000

+9 Volts

I

Heater

Gnd

AD534_b.pdf
Piezoresistance is defined as the change in electrical resistance of a solid when subjected to stress. The piezorestivity coefficient is $\Pi$ and a typical value may be $1 \times 10^{-10} \text{ cm}^2/\text{dyne}$.

The fractional change in resistance $\Delta R/R$ is given by:

$$\Delta R/R = \Pi \sigma$$

where $\sigma$ is the stress. Other references use the gage factor $GF$ to describe the piezoresistive effect where

$$\Delta R/R = (\Delta L/L) / GF$$
Example: Find the maximum stress in a simple polysilicon cantilever with the following parameters. Y\text{max} = 1 \, \mu m, b=4 \, \mu m, h=2\mu m, L=100 \, \mu m

\[ \sigma_{x=0} = 5.6e7 \, \text{newton/m}^2 = 5.6e8 \, \text{dyne/cm}^2 \]

From example in mem_mech.ppt

Continue Example: What is the change in resistance given \( \Pi = 1e-10 \, \text{cm}^2/\text{dyne} \)

\[ \Delta R/R = \Pi \sigma = (1e-10 \, \text{cm}^2/\text{dyne})/(5.6e8 \, \text{dyne/cm}^2) = 5.8\% \]
CALCULATION OF EXPECTED OUTPUT VOLTAGE FROM A PIEZORESISTIVE PRESSURE SENSOR

The equation for stress at the center edge of a square diaphragm (S.K. Clark and K. Wise, 1979)

\[ \text{Stress} = 0.3 \frac{P}{L/H^2} \]

where \( P \) is pressure, \( L \) is length of diaphragm edge, \( H \) is diaphragm thickness

For a 3000µm opening on the back of the wafer the diaphragm edge length \( L \) is

\[ 3000 - 2 \left( \frac{500}{\tan 54^\circ} \right) = 2246 \, \mu m \]
CALCULATION OF EXPECTED OUTPUT VOLTAGE

Stress = 0.3 P (L/H)^2

If we apply vacuum to the back of the wafer that is equivalent to and applied pressure of 14.7 psi or 103 N/m^2
P = 103 N/m^2
L = 2246 µm
H = 25 µm

Stress = 2.49E8 N/m^2

Hooke’s Law: Stress = E Strain where E is Young’s Modulus
σ = E ε

Young’s Modulus of silicon is 1.9E11 N/m^2
Thus the strain = 1.31E-3 or .131%
The sheet resistance (Rhos) from 4 point probe is 61 ohms/sq
The resistance is R = Rhos L/W
For a resistor R3 of L=350 µm and W=50 µm we find:
R3 = 61 (350/50) = 427.0 ohms

R3 and R2 decrease as W increases due to the strain
assume L is does not change, W’ becomes 50+50x0.131%
W’ = 50.0655 µm
R3’ = Rhos L/W’ = 61 (350/50.0655) = 426.4 ohms

R1 and R4 increase as L increases due to the strain
assume W does not change, L’ becomes 350 + 350x0.131%
R1’ = Rhos L’/W = 61 (350.459/50) = 427.6 ohms
CALCULATION OF EXPECTED OUTPUT VOLTAGE

No stress
Vo2-Vo1 = 0

Vo1=2.5v
Vo2=2.5v

With stress
Vo2-Vo1 = 0.007v
=7 mV
**CAPACITORS**

Symbol:

\[
I = C \frac{dV}{dt}
\]

Capacitors are used to store charge in memory cells, build filters, oscillators, integrators and many other applications in electronics.

\[
C = \varepsilon_0 \varepsilon_r \text{Area} / d
\]

\[
\varepsilon_0 = 8.85 \text{E}-14 \text{ F/cm}
\]

\[
\varepsilon_r \text{air} = 1
\]

\[
\varepsilon_r \text{SiO2} = 3.9
\]
CAPACITOR SENSOR

\[ C = \varepsilon_0 \varepsilon_r \text{Area} / d \]

Move one plate relative to the other results in a change in capacitance

Aluminum diaphragm pressure sensor
Energy stored in a parallel plate capacitor $W$
with area $A$ and space between plates of $d$

$$W = QV = CV^2$$
since $Q = CV$

The energy stored in a capacitor can be equated to the force times distance between the plates

$$W = Fd \quad \text{or} \quad F = \frac{W}{d}$$

$$F = \frac{\varepsilon_0 \varepsilon_r AV^2}{2d^2}$$

$\varepsilon_0 = \text{permittivity of free space} = 8.85\times10^{-12} \text{ Farads/m}$

$\varepsilon_r = \text{relative permittivity (for air $\varepsilon_r = 1$)}$
ELECTROSTATIC FORCE EXAMPLE

Example: 100 µm by 100 µm parallel plates
space = 1 µm, voltage = 10 V
Find the force of attraction between the two plates

\[ F = \frac{\varepsilon_0 \varepsilon_r AV^2}{2d^2} \]

\[ F = \frac{(8.85 \times 10^{-12})(1)(100 \times 10^{-6})(100 \times 10^{-6})(10)^2}{2(10^{-6})^2} \]

\[ F = 4.42 \times 10^{-6} \text{ newtons} \]
MEMS CANTILEVER WITH ELECTROSTATIC ACTUATOR

As the voltage is increased the electrostatic force starts to pull down the cantilever. The spring constant opposes the force but at the same time the gap is increased and the force increases. The electrostatic force increases with 1/d² so eventually a point is reached where it is larger than the spring and the cantilever snaps down all the way. The voltage has to be reduced almost to zero to release the cantilever.
Example: Plot the displacement versus current assuming poly sheet resistance of 200 ohms/sq, a piezoresistance coefficient of 1e-10 cm/dyne, d=1.5 µm, h=2µm and other appropriate assumptions.
SOLUTION FOR \( y \) VERSUS \( I \)

\[
V = I(R + \pi \sigma)
\]

\[
F = \frac{\varepsilon \sigma \varepsilon r AV^2}{2d^2}
\]

\[
F = \frac{Y_{\text{max}} 3 E bh^3}{12L^3}
\]

\[
\sigma_{x=0} = \frac{12FL}{2bh^2}
\]
Electrostatic movement parallel to wafer surface

From Jay Zhao
CALCULATION OF DISPLACEMENT VS VOLTAGE

\[ C = \varepsilon r \varepsilon_0 \, t \, \frac{L}{d} \]

\[ F = \varepsilon r \varepsilon_0 \, t \, \frac{V^2}{2 \, d} \]
COMBINED SPRING ELECTROSTATIC DRIVE AND CAPACITIVE OR PIEZORESISTIVE READ OUT
Figure 1: Top down CAD design of single axis Torsional Mirror

$$F = \frac{(m_m m_{coil})}{\mu_o z} = \frac{\mu_0}{4\pi} \left( \frac{2\pi R^2 I}{\left(z^2 + R^2\right)^{3/2}} \right) \left(LW\right)^2 B_m$$
Magnetic flux density of a permanent magnet \( B \) is given by the manufacturer in units of weber/sq.meter or Tesla. (some of the magnets we use in MEMS are 2mm in diameter and have \( B=0.5 \) Tesla)

Magnetic flux density \( B \) in the center of a coil

\[
B_{\text{coil}} = N \times B_{\text{loop}}
\]

where

\[
B_{\text{loop}} = \frac{\mu_0}{4\pi} \left( \frac{2\pi R^2 I}{z^2 + R^2} \right)^{3/2}
\]

The magnetic pole strength is \( m \) (webers) = \( B \times A \) where \( A \) is the pole area.

The force between two poles is

\[
\text{Force} = \frac{m_1 m_2}{\mu_0 z^2}
\]

\[
\mu_0 = 4\pi \times 10^{-7} \text{ w}^2/\text{Nm}^2
\]

http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/magfie.html
MAGNETIC DEVICES

Force on a straight conductor in a uniform magnetic field.

\[ F = I \cdot L \times B \]

Force on a coil with current \( I \) in a uniform magnetic field

\[
F = \left( \frac{m_m m_{coil}}{\mu_o} \right) = \frac{\mu_0}{4\pi} \left( \frac{2\pi R^2 I}{(z^2 + R^2)^{3/2}} \right) (LW)^2 B_m
\]
at center of a solenoid

Magnetic Flux Density (Tesla)

\[ B = \mu_0 \mu_r NI/L \]

where \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \)
\( \mu_r = 600 \) for Ni

Energy = \( W_m = B^2/2\mu_0\mu_r \)

or

\[ W_m = (\mu_0\mu_r/2) (NI/L)^2 \]
MAGNETIC SOLENOID

The energy stored in a small slice (dx) in the core of area (A) is

\[ \frac{W_m}{dx} = \left( \frac{\mu_0 \mu_r}{2} \right) (NI/L)^2 A \]

the energy difference to remove the core by dx is dWm

\[ dW_m = \frac{W_m}{dx} - \frac{W_m'}{dx} \]

the change in energy equals force times change in distance

\[ Fdx = dW_m \]

\[ F = \frac{dW_m}{dx} = \left( \frac{\mu_0}{2} \right) (\mu_r - 1) (NI/L)^2 A \]
MAGNETIC SOLENOID

Example: Given a nickel core solenoid of length 200 µm with 25 turns and 0.1 amp of current. The cross-sectional area is 40 µm by 2µm. Calculate the force needed to move the core.

\[ F = \frac{\mu_0}{2}(\mu_r-1) \left(\frac{NI}{L}\right)^2A \]

\[ F = \left(4\pi \times 10^{-7}/2\right)(600-1)\left[\frac{25(0.1)}{200 \times 10^{-6}}\right]^2(40 \times 10^{-6})(2 \times 10^{-6}) \]

\[ = 4.7 \times 10^{-6} \text{ Newton} \]
MAGNETIC - SOLENOID

2nd layer poly

40 µm

100 x100 µm pads

2nd layer poly

40 µm

100 x100 µm pads
EXAMPLE - SOLENOID

N: 20, P: 84μm, W: 68μm
Substrates: both Si and glass

500μm

N: number of turns
Copper bridge

50μm P
Copper bottom
MAGNETIC FIELD SENSORS

Lateral Magnetotransistor
MOS Magnetotransistor

\[ F = q(v \times B) \]

See UGIM 1999
Add Picture
A piezoelectric material will exhibit a change in length in response to an applied voltage. The reverse is also possible where an applied force causes the generation of a voltage. Single crystal quartz has been used for piezoelectric devices such as gas grill igniters and piezoelectric linear motors. Thin films of various materials (organic and inorganic) exhibit piezoelectric properties. ZnO films 0.2 μm thick are sputtered and annealed 25 min, 950°C giving piezoelectric properties. Many piezoelectric materials also exhibit pyroelectric properties (voltage - heat).
SEEBECK EFFECT

When two dissimilar conductors are connected together a voltage may be generated if the junction is at a temperature different from the temperature at the other end of the conductors (cold junction). This is the principal behind the thermocouple and is called the Seebeck effect.

\[
\Delta V = \alpha_1(T_{\text{cold}} - T_{\text{hot}}) + \alpha_2(T_{\text{hot}} - T_{\text{cold}}) = (\alpha_1 - \alpha_2)(T_{\text{hot}} - T_{\text{cold}})
\]

Where \(\alpha_1\) and \(\alpha_2\) are the Seebeck coefficients for materials 1 and 2.
PELTIER EFFECT

Heat pump device that works on the gain in electron energy for materials with low work function and the loss in energy for materials with higher work function. Electrons are at higher energy (lower work function) in n-type silicon.
PELTIER HEAT PUMP

Ferrotec America Corp
1050 Perimeter Rd, #202
Manchester, NH 03103
(603) 626-0700

Single Stage Coolers: Imax = 3.0 Amps

<table>
<thead>
<tr>
<th>TE MODULE NUMBER</th>
<th>Code</th>
<th>Qmax (W)</th>
<th>Vmax (V)</th>
<th>DTmax (°C)</th>
<th>Type</th>
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<td>2.3</td>
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<td>29.0</td>
<td>17.6</td>
<td>72</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Configuration A

Type 1

Type 2

Type 3

Rochester Institute of Technology
Microelectronic Engineering

© March 15, 2008 Dr. Lynn Fuller, Motorola Professor
I = I_o \, e^{(-\beta \phi z)}

where
Io = scaling factor
\beta = conversion factor (\sim 10.25 \text{ eV}^{-0.5}/\text{nm})
\phi = tunnel barrier height in eV (\sim 0.5\text{eV})
z = tip to surface separation (100 \text{Å})
INTEGRATED DIODES

p+ means heavily doped p-type
n+ means heavily doped n-type
n-well is an n-region at slightly higher doping than the p-wafer

Note: there are actually two pn junctions, the well-wafer pn junction should always be reverse biased
**DIODE TEMPERATURE DEPENDENCE**

\[
Id = I_s \left[\exp \left(\frac{q V_D}{KT}\right) - 1\right]
\]

Neglect the \(-1\) in forward bias, Solve for \(V_D\)

\[
V_D = \frac{KT}{q} \ln \left(\frac{Id}{I_s}\right) = \left(\frac{KT}{q}\right) \left(\ln(Id) - \ln(Is)\right)
\]  

\text{eq 1}

Take \(dV_D/dT\): note \(Id\) is not a function of \(T\) but \(Is\) is

\[
dV_D/dT = \left(\frac{KT}{q}\right) \left(d\ln(Id)/dT - d\ln(Is)/dT\right) + K/q \left(\ln(Id) - \ln(Is)\right)
\]

\text{zero}

\[
VD/T \text{ from eq 1}
\]

Rewritten

\[
dV_D/dT = VD/T - \left(\frac{KT}{q}\right) \left((1/Is) dIs/dT\right)
\]

\text{eq 2}

Now evaluate the second term, recall

\[
Is = qA \left(\frac{Dp}{(LpNd)} + \frac{Dn}{(LnNa)}\right)ni^2
\]

Note: \(Dn\) and \(Dp\) are proportional to \(1/T\)
and \( n_i^2(T) = A T^3 e^{-qE_g/KT} \)

This gives the temperature dependence of \( I_s \)

\[
I_s = C T^2 e^{-qE_g/KT}
\]

eq 3

Now take the natural log

\[
\ln I_s = \ln (C T^2 e^{-qE_g/KT})
\]

Take derivative with respect to \( T \)

\[
\frac{1}{I_s} \frac{d}{dT}(I_s) = \frac{1}{I_s} \frac{d}{dT}(C T^2 e^{-qE_g/KT}) = \frac{d}{dT}(C T^2 e^{-qE_g/KT})
\]

\[
= (1/I_s) [C T^2 e^{-qE_g/KT}(qE_g/KT^2) + (C e^{-qE_g/KT})2T]
\]

\[
= (1/I_s) [I_s(qE_g/KT^2) + (2I_s/T)]
\]

Back to eq 2

\[
\frac{dV_D}{dT} = \frac{V_D}{T} - \left(\frac{KT}{q}\right) [(qE_g/KT^2) + (2/T)]
\]

\[
\frac{dV_D}{dT} = \frac{V_D}{T} - \frac{E_g}{T} - \frac{2K}{q}
\]
EXAMPLE: DIODE TEMPERATURE DEPENDENCE

\[ \frac{dV_D}{dT} = \frac{V_D}{T} - \frac{E_g}{T} - \frac{2K}{q} \]

Silicon with \( E_g \approx 1.2 \text{ eV} \), \( V_D = 0.6 \text{ volts} \), \( T=300 \text{ °K} \)

\[ \frac{dV_D}{dT} = \frac{0.6}{300} - \frac{1.2}{300} - \frac{2(1.38E-23)}{1.6E-19} \]

\[ = -2.2 \text{ mV/°} \]
space charge layer

<table>
<thead>
<tr>
<th>Electron and hole pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phosphorous donor atom and electron</td>
</tr>
<tr>
<td>Ionized Immobile Phosphorous donor atom</td>
</tr>
<tr>
<td>Ionized Immobile Boron acceptor atom</td>
</tr>
<tr>
<td>Boron acceptor atom and hole</td>
</tr>
</tbody>
</table>
PHOTODIODE

- Diagram of a photodiode with labels for V, I, n, p, and arrows indicating flow.
- Graph showing absorption coefficient as a function of wavelength for silicon.
- Legend indicating different light conditions: No Light, More Light, Most Light.
CHARGE COLLECTION IN MOS STRUCTURES

\[ E = h\nu = \frac{hc}{\lambda} \]

\[ h = 6.625 \text{ e}-34 \text{ J/s} \]
\[ = \left(\frac{6.625 \text{ e}-34}{1.6\text{e}-19}\right) \text{ eV/s} \]

\[ E = 1.55 \text{ eV (red)} \]
\[ E = 2.50 \text{ eV (green)} \]
\[ E = 4.14 \text{ eV (blue)} \]

Electron and hole pair

deployment region

\(\lambda_1\)
\(\lambda_2\)
\(\lambda_3\)
\(\lambda_4\)

Thin poly gate

P-type
LIGHT EMITTING DIODES (LEDs)

Electron concentration vs distance

Hole concentration vs distance

P-side

N-side

In the forward biased diode current flows and as holes recombine on the n-side or electrons recombine on the p-side, energy is given off as light, with wavelength appropriate for the energy gap for that material. \( \lambda = \frac{h \cdot c}{E} \)

\( h = \text{Plank’s constant} \)

\( c = \text{speed of light} \)
REFERENCES

HOMEWORK – MEMS ELECTRICAL

1. Calculate the voltage needed to pull down a polysilicon cantiliver by electrostatic force. Make appropriate assumptions for dimensions.
2. Calculate the number of fingers needed in an electrostatic comb drive to create a force of 10 micro newtons. Make appropriate assumptions for dimensions.
3. What coil current is needed to create a force of 10 micro newtons in a magnetic field of 0.5 Tesla. Make appropriate assumptions for dimensions.
4. In a thermocouple made of aluminum on n+ poly what voltage will be generated for a temperature difference of 70 °C?
5. A diode is used as a temperature sensor and is forward biased with a 1.5 Volt battery in series with a 10Kohm resistor. If the device is used to measure body temperature (nominal 98.6 °F) how much change in voltage across the diode if someone had a temperature of 102.6°F.