Scaling Laws, Actuators, and Power in Miniaturization

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INTRODUCTION

As objects shrink, the ratio of surface area to volume increases, rendering surface forces more important. For example, if we build two bridges geometrically similar, the larger will be the weaker of the two and will be so in the ratio of their linear dimensions (l). The strength of the iron girders in the bridges varies with the square of the linear dimension (l^2), but the weight of the whole structure varies with the cube of its linear dimension (l^3). The larger the structure, the more severe the strain becomes. By reducing the size of a device, the structural stiffness generally increases relative to inertially imposed loads. Another striking example is that of surface tension. The mass of a liquid in a capillary tube, and hence the weight, scales as l^3 and decreases more rapidly than the surface tension, which scales as l as the system becomes smaller. That is why it is more difficult to empty liquids from a capillary compared to spilling coffee from a cup.
INTRODUCTION

Without relying on complicated math we can already appreciate that, as the scale of structures decreases, so does the importance of phenomena that vary with the largest power of the linear dimension $l$: gravity ($l^3$), inertia ($l^3$), magnetism ($l^2$, $l^3$ or $l^4$, depending on the exact configuration), flow ($l^4$), and thermal emission ($l^2$ to $l^4$). Phenomena that are more weakly dependent on size dominate in small dimensions: electrostatics ($l^2$), friction ($l^2$), surface tension ($l$), diffusion ($l^{1/2}$), and van der Wall’s forces ($l^{1/4}$)
SCALING IN ELECTROSTATICS

Energy (E) stored in a parallel plate capacitor (C),

\[ E = \frac{1}{2} CV^2 \]

where \( V \) is the capacitor voltage

The capacitance \( C = \varepsilon_0 \varepsilon_r \frac{wl}{d} \)

where \( w \) and \( l \) give the plate area (A)
and \( d \) is the plate spacing

The maximum voltage depends on the dielectric breakdown electric field, \( E_{\text{max}} = \frac{V}{d} \)

Combining \[ E = \frac{1}{2} \varepsilon_0 \varepsilon_r \frac{wl}{d} E_{\text{max}}^2 \]
thus \( E \) stored is proportional to dimension cubed \((l^3)\)
Force between two parallel plates, capacitor (C) of area (A)

\[ F = \frac{\varepsilon_0 \varepsilon_r w l V^2}{2d^2} \]

The maximum voltage depends on the dielectric breakdown electric field, \( E_{\text{max}} = V/d \)

Combining \( F = \frac{1}{2} \varepsilon_0 \varepsilon_r w l E_{\text{max}}^2 \)

thus \( F \) is proportional to dimension squared \((l^2)\)
GENERAL SCALING LAWS

Small things are fast
Small things are strong
Small things are not affected by gravity much compared to electrostatic forces or friction
Small things do not provide much Torque or Power
Small things are dominated by Van der Waals Force, surface tension and diffusion

<table>
<thead>
<tr>
<th>Force</th>
<th>Exponent</th>
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<tbody>
<tr>
<td>Time</td>
<td>$l^0$</td>
</tr>
<tr>
<td>van der Waals Force</td>
<td>$l^{1/4}$</td>
</tr>
<tr>
<td>Diffusion</td>
<td>$l^{1/2}$</td>
</tr>
<tr>
<td>Distance</td>
<td>$l^1$</td>
</tr>
<tr>
<td>Velocity</td>
<td>$l^1$</td>
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<tr>
<td>Surface Tension</td>
<td>$l^1$</td>
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<tr>
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<tr>
<td>Muscle Force</td>
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<tr>
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<td>Thermal Losses</td>
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<tr>
<td>Piezo-electricity</td>
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<tr>
<td>Mass</td>
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<tr>
<td>Gravity</td>
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<tr>
<td>Torque</td>
<td>$l^3$</td>
</tr>
<tr>
<td>Power</td>
<td>$l^3$</td>
</tr>
</tbody>
</table>
REFERENCES

1. Derive an isomorphic scaling relationship for the following quantities:
   a) maximum temperature in a resistance heater
   b) maximum vertical liquid level in a capillary tube
   c) any other quantity