Discrete Math I
Quiz #1 (12/8/11)

Instructions: Provide all steps necessary to solve the problem. Simplify your answer as much as possible. Additionally, clearly indicate the value or expression that is your final answer!

Note that there are problems on both sides.

1. What is the contrapositive of the statement “If it is cloudy, then I will study”?

2. Evaluate the bitwise expression \((1\ 1011 \text{ OR } 0\ 1010) \text{ XOR } 0\ 1000\).

3. Construct a truth table for the compound proposition \(p \land (p \to q) \to \neg q\).
4. Find a simple proposition with the truth table below. Your answer should involve $p$ and $q$ and one or more of the following operations: $\neg$, $\land$, $\lor$, $\rightarrow$.

\[
\begin{array}{|c|c|c|}
\hline
p & q & ? \\
\hline
T & T & F \\
T & F & F \\
F & T & T \\
F & F & F \\
\hline
\end{array}
\]

Extra Credit: The *Numerical Dictionary* is a special book which contains all the integers written in English (no “and”) in alphabetical order. Determine (a) the first entry, (b) the last entry, and (c) the next to last entry.
Discrete Math I – Solutions to Quiz #1

1. “If I will not study, then it is not cloudy.”

Note: Some students were tempted to write the contrapositive of
“If \( p \), then \( q \)”
as
“Not \( q \), if not \( p \)”
which is equivalent to
“If not \( p \), then \( q \)”.  
This is the inverse, not the contrapositive of the statement.

2. 1 0011

3. 

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( [p \wedge (p \rightarrow q)] \rightarrow \neg q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
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<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

4. Two answers were \( \neg p \wedge q \) and \( \neg (q \rightarrow p) \). There are many more that work, but these are the shortest.

Extra Credit: Proposed by Richard Hess and published in the *Journal of Recreational Mathematics* as problem 2608.

(a) Eight

(b) Zero

(c) An answer that is worth partial credit is
“Two trillion two thousand two hundred two” (2,000,000,002,202).
The better answer is
“Two vigintillion two undecillion two trillion two thousand two hundred two”
(2,000,000,000,000,000,000,000,002,000,000,000,000,000,000,000,000,002,000,000,002,202).
Discrete Math I – Quiz #1

<table>
<thead>
<tr>
<th>P</th>
<th>Answer/Solution</th>
<th>A %</th>
<th>M %</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>“If I will not study, then it is not cloudy.”</td>
<td>85</td>
<td>100</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>1 0011</td>
<td>98</td>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>$p \land (p \to q) \to \neg q$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$p$</td>
<td>$q$</td>
<td>$\neg q$</td>
<td></td>
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<td>T</td>
<td>T</td>
<td>F</td>
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<td>F</td>
<td>T</td>
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<tr>
<td>4</td>
<td>Answer #1: $\neg p \land q$</td>
<td>90</td>
<td>100</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Answer #2: $\neg (q \to p)$</td>
<td></td>
<td></td>
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<tr>
<td>Overall</td>
<td></td>
<td>93</td>
<td>99</td>
<td>16</td>
</tr>
</tbody>
</table>

**EC (a)**: Eight {23 students received credit} 0.25

**EC (b)**: Zero {18 students received credit} 0.25

**EC (c)**: Two trillion two thousand two hundred two {2 students received credit} 1