1. Given two sets $X$ and $Y$, use a Venn diagram to determine which of the following are always true: $X \subseteq Y$, $X \subseteq Y$, $X = Y$, $Y \subseteq X$, $Y \subseteq X$. There may be more than one correct answer (or, perhaps, none at all).

(a) $X = A \cup B$ and $Y = A \cup (B - A)$

Circle one or more: $X \subseteq Y$, $X \subseteq Y$, $X = Y$, $Y \subseteq X$, $Y \subseteq X$, NONE

(b) $X = A \cup (B \cap C)$ and $Y = (A \cup B) \cap C$

Circle one or more: $X \subseteq Y$, $X \subseteq Y$, $X = Y$, $Y \subseteq X$, $Y \subseteq X$, NONE

2. Find the following values.

(a) $\left[ \frac{10}{3} \right]$

(b) $\left[ -\frac{10}{3} \right]$

(c) $\left\lfloor \frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} \right\rfloor + \frac{3}{2}$
3. Let \( f : \{1,2,3,4,5\} \rightarrow \{a,b,c,d,e\} \) be a function defined by \( f = \{(1,a), (2,b), (3,d), (4,e), (5,b)\} \).

(a) Find the domain of \( f \).

(b) Find the codomain of \( f \).

(c) Find the range of \( f \).

(d) Is \( f \) one-to-one?

(e) Is \( f \) onto?

4. Let \( S = \{-1, 0, 1, 3, 9\} \). Let \( f : \mathbb{Z} \rightarrow \mathbb{Z} \) be the function given below.

(a) Calculate \( f(S) \) if \( f(x) = 2 \).

(b) Calculate \( f(S) \) if \( f(x) = \lceil \frac{x}{4} \rceil \).

5. Let \( A = \{a,b\} \) and \( B = \{x,y,z\} \). Determine the number of different functions that can be defined from \( A \) to \( B \)?

Extra Credit: Determine all real numbers \( x \) for which \( x^2 - \lfloor x \rfloor = \frac{1}{2} \).
Discrete Math I – Solutions to Quiz 4

1. 
   (a) The sets are equal, so \( X \subseteq Y, X = Y, \) and \( Y \subseteq X. \)
   
   
   
   
   (b) Thus, \( Y \subseteq X \) only.

2. 
   (a) \( \left\lfloor \frac{10}{3} \right\rfloor = \left\lfloor 3.33 \ldots \right\rfloor = 3 \)
   
   (b) \( \left\lceil -\frac{10}{3} \right\rceil = \left\lceil -3.33 \ldots \right\rceil = -4 \)
   
   (c) \( \left\lfloor \frac{1}{2} \right\rfloor + \left\lceil \frac{1}{2} \right\rceil + \frac{3}{2} = \left\lfloor 0 + 1 + \frac{3}{2} \right\rfloor = \left\lfloor 2.5 \right\rfloor = 3 \)

3. 
   (a) The domain is \( \{1,2,3,4,5\}. \)
   
   (b) The codomain is \( \{a,b,c,d,e\}. \)
   
   (c) The range is \( \{a,b,d,e\}. \)
   
   (d) No, since \( f(2) = f(5) = b. \)
   
   (e) No, since none of the elements in the domain map to \( c. \)

4. Note that the definition of the image of a set was given at the top of the first page.
   (a) Since \( f(-1) = f(0) = f(1) = f(3) = f(9) = 2, \) then \( f(S) = \{2\}. \)
   (b) Since \( f(-1) = -1, f(0) = f(1) = f(3) = 0, \) and \( f(9) = 2, \) then \( f(S) = \{-1,0,2\}. \)

5. There are 9 functions. Just to show you, here are all of them.
   
   \[
   \begin{align*}
   g_1 &= \{(a,x), (b,x)\} & g_4 &= \{(a,y), (b,x)\} & g_7 &= \{(a,z), (b,x)\} \\
   g_2 &= \{(a,x), (b,y)\} & g_5 &= \{(a,y), (b,y)\} & g_8 &= \{(a,z), (b,y)\} \\
   g_3 &= \{(a,x), (b,z)\} & g_6 &= \{(a,y), (b,z)\} & g_9 &= \{(a,z), (b,z)\}
   \end{align*}
   \]
Extra Credit: There are exactly two solutions: $\sqrt{\frac{1}{2}}$ and $\sqrt{\frac{1}{2}}$. To see that there are not any other solutions, see the following graph of $f(x) = x^2 - \lfloor x \rfloor$ drawn in MAPLE. (Ignore the vertical lines; this is how MAPLE interprets discontinuities.)

The curve crosses the line $y = \frac{1}{2}$ in exactly two points.
### Discrete Math I – Quiz #4

<table>
<thead>
<tr>
<th>P</th>
<th>Answer/Solution</th>
<th>A %</th>
<th>M %</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>The sets are equal, so $X \subseteq Y$, $X = Y$, and $Y \subseteq X$.</td>
<td>87</td>
<td>90</td>
<td>3</td>
</tr>
<tr>
<td>1b</td>
<td>$Y \subseteq X$ only.</td>
<td>75</td>
<td>88</td>
<td>3</td>
</tr>
<tr>
<td>2a</td>
<td>3</td>
<td>92</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>2b</td>
<td>$-4$</td>
<td>92</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>2c</td>
<td>3</td>
<td>87</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>3a</td>
<td>${1,2,3,4,5}$</td>
<td>98</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>3b</td>
<td>${a,b,c,d,e}$</td>
<td>85</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>3c</td>
<td>${a,b,d,e}$</td>
<td>88</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>3d</td>
<td>No, since $f(2) = f(5) = b$.</td>
<td>83</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>3e</td>
<td>No, since none of the elements in the domain map to $c$.</td>
<td>78</td>
<td>100</td>
<td>1</td>
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<tr>
<td>4a</td>
<td>$f(S) = {2}$</td>
<td>87</td>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td>4b</td>
<td>$f(S) = {-1,0,2}$</td>
<td>82</td>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>Nine functions</td>
<td>60</td>
<td>55</td>
<td>2</td>
</tr>
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<td></td>
<td><strong>Overall</strong></td>
<td><strong>82.5</strong></td>
<td><strong>83.3</strong></td>
<td><strong>20</strong></td>
</tr>
<tr>
<td>ec</td>
<td>There are exactly two solutions: $\sqrt{\frac{1}{2}}$ and $\sqrt{\frac{3}{2}}$.</td>
<td></td>
<td></td>
<td>1.5</td>
</tr>
</tbody>
</table>

{12 students received some or all of the points available}