1. Pick the best choice (10 points)
__b__ A. A population has mean 5 and standard deviation 10. A random sample of 100 observations is obtained. The sample mean \( \bar{x} \) has a. mean 5 and standard deviation 10 b. mean 5 and standard deviation 1 c. mean 0.5 and standard deviation 1 d. mean 0.5 and standard deviation 10 e. mean 5 and standard deviation 100.

__d__ B. A sample of size 100 is drawn from a population with known standard deviation 10. The sample mean \( \bar{x} = 10.0 \). A 95% confidence interval is a. (8.355, 10.1645) b. (7.674, 12.326) c. (-9.6, 39.6) d. (8.04, 11.196) e. none of the choices a-d.

__d__ C. The p value of a statistical test is .035. This means that you a. cannot reject the null hypothesis for any value of \( \alpha \) b. would reject the null hypothesis at \( \alpha = .01 \) c. would reject the null hypothesis at both \( \alpha = .05 \) and \( \alpha = .01 \) d. reject the null hypothesis at \( \alpha = .05 \) but fail to reject the null hypothesis at \( \alpha = .01 \) e. fail to reject the null hypothesis at \( \alpha = .05 \).

__a__ D. A 90% confidence interval a. has probability .90 of containing the true population parameter b. always contains the true population parameter c. is wider than a 95% confidence interval d. rejects all of the null hypotheses whose numerical value belongs to the interval e. has none of the characteristics mentioned in a-d.

__c__ E. A test of hypothesis of \( H_0: \mu = 10 \) vs. \( H_1: \mu \neq 10 \) for a large sample yields a calculated value of \( Z=2.05 \). The p value is approximately a. 0.05 b. 0.02 c. 0.04 d. 0.01 e. 0.10

2. USE THE CENTRAL LIMIT THEOREM TO DO THIS PROBLEM
1. The shape of the distribution of the time required to get an oil change at a 10-minute oil change facility is unknown. However records indicate that the mean time for an oil change is 11.4 minutes and the standard deviation for oil-change time is 3.2 minutes. Consider a random sample of 64 oil changes.

A. What is the mean and the standard deviation of the sample mean oil change time? (5 points)
\[
\mu_\bar{x} = 11.4, \sigma_\bar{x} = \frac{3.2}{\sqrt{64}} = 0.4
\]

B. What is the probability that the sample mean oil change time is at most 11 minutes? (10 points)
\[
Z = \frac{11-11.4}{0.4} = -1, P(Z \leq -1) = .1587 \text{ or normalcdf} (-10^99, 11.114, 4) = .1587
\]
3. Every Monday the Energy Information Administration (EIA) determines the national average gasoline price by collecting retail prices for gasoline from a sample of 900 retail gasoline outlets from across the nation. On July 14, 2008, the EIA reported the national average retail price for regular grade gasoline to be $4.113 per gallon.

A. Assuming that the population standard deviation is $\sigma = 0.110$ per gallon construct a 99% confidence interval for the national mean price per gallon for regular-grade gasoline on July 14, 2008. (10 points)

$$
(\bar{x} - \frac{z_{\alpha/2}\sigma}{\sqrt{n}}, \bar{x} + \frac{z_{\alpha/2}\sigma}{\sqrt{n}})
$$

$$
(4.113 - \frac{2.576(0.110)}{\sqrt{900}}, 4.113 + \frac{2.576(0.110)}{\sqrt{900}})
$$

$$(4.103, 4.122)$$

B. How large a sample is needed so that the error bound is at most .005? (8 points)

$$
n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2
$$

$$
n = \left(\frac{2.576(0.110)}{0.005}\right)^2 = 3211.7
$$

$$
n = 3212$$
4. The following data represent the age in weeks at which babies first crawl based on a survey of 12 mothers conducted by Essential baby.

<table>
<thead>
<tr>
<th>52</th>
<th>30</th>
<th>44</th>
<th>35</th>
<th>39</th>
<th>26</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>37</td>
<td>56</td>
<td>26</td>
<td>39</td>
<td>28</td>
</tr>
</tbody>
</table>

A. Construct a 95% confidence interval for the true mean age at which a baby first crawls. (15 points)

\[
(\bar{x} - \frac{t_{\alpha/2} \sigma}{\sqrt{n}}, \bar{x} + \frac{t_{\alpha/2} \sigma}{\sqrt{n}})
\]

\[
(38.35 - \frac{2.201(10)}{\sqrt{12}}, 38.35 + \frac{2.201(10)}{\sqrt{12}}) \quad \bar{x} = 38.25, \quad s = 10
\]

(31.996, 44.704)

B. Write a sentence that interprets the interval that you obtained in part A. (4 points)

The probability that the interval contains the true mean age at which a baby first crawls is .99.

5. Calcium is essential to tree growth because it promotes the formation of wood and maintains cell walls. In 1990 the concentration of calcium in precipitation in Chautauqua, New York was 0.11 milligrams per liter. In 2007 a random sample of 10 precipitation dates yielded a sample mean of 0.146. The objective is to perform a test of hypothesis to determine if the calcium level has increased for 1990 to 2007 assuming a known standard deviation of 0.08.

A. What would be the null and the alternative hypothesis for this test? (2 points)

\[H_0: \mu = 0.11\]
\[H_1: \mu > 0.11\]

B. What are the assumptions that this test is performed under? (2 points)

Normal population with known standard deviation.

C. What is the region of rejection if the test is conducted at \(\alpha = .05\)? (2 points)

\[Z > 1.645\]
D. Compute the value of the test statistic (4 points)

\[ Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \]

\[ Z = \frac{.146 - .11}{.08 / \sqrt{10}} = 1.423 \]

E. Would you reject or fail to reject \( H_0 \) at \( \alpha = .05 \)? (2 points)

Cannot reject \( H_0 \)

F. What conclusion to you come to about the change in calcium concentration based on your answers of A-E? (2 points)

There is not enough evidence to conclude that there is a significant change in calcium concentration.

G. Find the p value for this test. (2 points)

\[ P(Z>1.42)=.0778 \]

6. According to the Boston Rental Exchange the mean monthly rental cost of a one bedroom apartment in the Cambridge area of Boston is $1400. A random sample of 12 apartments yields the following data.

<table>
<thead>
<tr>
<th>Rent in dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>1325</td>
</tr>
<tr>
<td>1609</td>
</tr>
<tr>
<td>1850</td>
</tr>
<tr>
<td>1600</td>
</tr>
<tr>
<td>1800</td>
</tr>
<tr>
<td>1500</td>
</tr>
<tr>
<td>1750</td>
</tr>
<tr>
<td>1475</td>
</tr>
<tr>
<td>1750</td>
</tr>
<tr>
<td>1450</td>
</tr>
<tr>
<td>1275</td>
</tr>
<tr>
<td>1150</td>
</tr>
</tbody>
</table>

The objective is to perform a test of hypothesis to determine if the mean rental is in fact different from $1400.

A. What would be the null and the alternative hypothesis for this test? (2 points)

\( H_0 \) \( \mu = 1400 \)

\( H_1 \) \( \mu \neq 1400 \)
B. Under what assumptions is this test performed?(2 points)

- Small sample
- Unknown sd
- Normal population

C. For $\alpha = 0.05$ what would be the region of rejection?(2 points)

$T > 2.201 \quad T < -2.201$

**One-Sample T: Rent**

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
<th>95% CI</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent</td>
<td>12</td>
<td>1544.5</td>
<td>222.1</td>
<td>64.1</td>
<td>(1403.4, 1685.6)</td>
<td>2.25</td>
<td>0.046</td>
</tr>
</tbody>
</table>

D. In the above Minitab output what is the calculated value of T and the p value?(2 points)

$T = 2.25 \quad p = 0.046$

E. Based on these values would you reject $H_0$ at $\alpha = 0.05$? Explain.(2 points)

Reject $H_0$ if $2.25 > 2.201 \quad p = 0.046 < 0.05$

F. Would you conclude that the rent might be different from $1400? Explain. (2 points)

Yes I might come to this conclusion. We rejected $H_0$ at $\alpha = 0.05$ indicating that the rent is significantly different from 1400.
7. In the recent election a candidate sampled 1000 candidates and found that 545 would vote for him.

A. Find a 98% confidence interval on the proportion of voters that favor this candidate. (5 points)

\[
p = \frac{X}{n} = \frac{545}{1000}
\]

\[
\left( \hat{p} - z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}, \hat{p} + z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \right)
\]

\[
(0.545 - 2.326 \sqrt{\frac{(0.545)(0.455)}{1000}}, 0.545 + 2.326 \sqrt{\frac{(0.545)(0.455)}{1000}})
\]

\[
(0.545 - 0.037, 0.545 + 0.037)
\]

\[
(0.508, 0.582)
\]

C. What sample size is needed to obtain a 98% confidence bound with width 0.05? (4 points)

\[
n = 0.25 \left( \frac{z_{\frac{\alpha}{2}}}{E} \right)^2
\]

\[
n = 0.25 \left( \frac{2.326}{0.05} \right)^2 = 541.02
\]

\[
n = 542
\]