SMAM 351 Homework 5 Solution

1. A. \( F(n) = \begin{cases} 
0 & n < 0 \\
0.7 & 0 \leq n < 1 \\
0.9 & 1 \leq n < 2 \\
1 & n > 2 
\end{cases} \)

Please note how the CDF is presented.

B. \( E(N) = 0(0.7) + 1(0.2) + 2(0.1) = 0.4 \)

\( E(N^2) = 0^2(0.7) + 1^2(0.2) + 2^2(0.1) = 0.6 \)

\( \text{Var}(N) = 0.6 - 0.16 = .44 \)

\( \text{sd}(N) = .6633 \)

C. \( R = 100,000N \)

\( E(R) = 100,000E(N) = 100,000(0.4) = $40,000 \)

\( \text{sd}(R) = 100,000\text{sd}(N) = 100,000(.6636) = $66,360 \)

D. Let \( N_1 \) be the number of sales the first day.

Let \( N_2 \) be the number of sales the second day.

The total number of sales is \( T = N_1 + N_2 \)

\( P(T = 0) = P(N_1 = 0)P(N_2 = 0) = (0.7)(0.7) = 0.49 \)

\( P(T = 1) = P(N_1 = 1)P(N_2 = 0) + P(N_1 = 0)P(N_2 = 1) = (0.2)(0.7) + (0.7)(0.2) = 0.28 \)

\( P(T = 2) = P(N_1 = 0)P(N_2 = 2) + P(N_1 = 1)P(N_2 = 1) + P(N_1 = 2)P(N_2 = 0) = (0.7)(0.1) + (0.2)(0.2) + (0.1)(0.7) = 0.18 \)

\( P(T = 3) = P(N_1 = 2)P(N_2 = 1) + P(N_1 = 1)P(N_2 = 2) = (0.1)(0.2) + (0.2)(0.1) = .04 \)

\( P(T = 4) = P(N_1 = 2)P(N_2 = 2) = (0.1)(0.1) = 0.01 \)

\[
\begin{array}{c|cccc}
T & 0 & 1 & 2 & 3 & 4 \\
g(t) & 0.49 & 0.28 & 0.18 & 0.04 & 0.01
\end{array}
\]

Part D was well done by many of the students.

2. A. Worksheet size: 100000 cells
   MTB > PDF 50;
   SUBC> Binomial 76 .64.
Probability Density Function
Binomial with \( n = 76 \) and \( p = 0.640000 \)

\[
\begin{array}{lcr}
  x & P( X = x) \\
  50.00 & 0.0911 \\
\end{array}
\]

B.
MTB > let c1=1
MTB > CDF 34;
SUBC> Binomial 76 .64.

Cumulative Distribution Function
Binomial with \( n = 76 \) and \( p = 0.640000 \)

\[
\begin{array}{lcr}
  x & P( X \leq x) \\
  34.00 & 0.0005 \\
\end{array}
\]

MTB > let c2=0.0005
MTB > let c3=c1-c2
MTB > print c3

Data Display

C3
0.9995

C. Worksheet size: 100000 cells
MTB > cdf 43 c6;
SUBC>
SUBC> binomial 76,.36.
MTB > cdf 29 c7;
SUBC> binomial 76, .36.
MTB > let c8=c6-c7
MTB > print c8

Data Display

C8
0.301704

Some people had used .64 instead of .36. The question called for the students who would reply that the quality of life was unsatisfactory. Some students did not hand in a computer printout. Use of the software was a requirement for this problem.
People who used Mathematica should use the Statistics packages. Use help to find out about them. Your printout should have looked something like this. Direct computation was not needed.

\[\text{\texttt{\textless\textless Statistics`DiscreteDistributions\texttt{\textgreater\textgreater} \textbackslash}}\]

\[\text{\texttt{b = BinomialDistribution[76, .64] \texttt{\textgreater\textgreater}}}\]
\[\text{\texttt{BinomialDistribution[76, 0.64] \texttt{\textgreater\textgreater}}}\]
\[\text{\texttt{PDF[b, 50] \texttt{\textgreater\textgreater}}}\]
\[\text{\texttt{0.0911161 \texttt{\textgreater\textgreater}}}\]
\[\text{\texttt{1 - CDF[b, 34] \texttt{\textgreater\textgreater}}}\]
\[\text{\texttt{0.999526 \texttt{\textgreater\textgreater}}}\]
\[\text{\texttt{c = BinomialDistribution[76, .36] \texttt{\textgreater\textgreater}}}\]
\[\text{\texttt{BinomialDistribution[76, 0.36] \texttt{\textgreater\textgreater}}}\]
\[\text{\texttt{CDF[c, 43] - CDF[c, 29] \texttt{\textgreater\textgreater}}}\]
\[\text{\texttt{0.301704 \texttt{\textgreater\textgreater}}}\]