1. Let $X$ be any random variable with mean 50 and standard deviation 5.

A. Find a lower bound on $P(37.5 < X < 62.5)$ using Chebychev’s Theorem

$5k = 12.5$

$k = 2.5$

Lower bound $= 1 - \frac{1}{(2.5)^2} = .84$

B. Find an upper bound on (1) $P(X > 60) + P(X < 40)$

The upper bound is .25

(2) $P(X > 60)$

The upper bound is also .25.

C. Find the value of the constant $c$ where

$P(|X - 50| \geq c) \leq 0.09$

$c = 5k$

$\frac{1}{k^2} = .09$

$.09k^2 = 1$

$k^2 = 11.11$

$k = 3.33$

$c = 3.33(5) = 16.67$

D. Assuming $X$ is normally distributed with mean 50 and standard deviation 5 find the exact probabilities for parts A and B.

$P(37.5 < X < 62.5) = P(-2.5 < Z < 2.5) = .9938 - .0062 = .9876$

$P(X > 60) = P \left( Z > \frac{60 - 50}{5} \right) = P(Z > 2) = .0228$

$P(X > 60) + P(X < 40) = .0456$
2. The life of a certain type of small motor is normally distributed with mean 10 years and standard deviation 1.5 years. The manufacturer will replace for free all motors that fail while under guarantee. He is only willing to replace at most 1.5% of the motors. How long a guarantee should he offer?

\[ P(X < c) = 0.015 \]
\[ P\left( Z < \frac{c - 10}{1.5} \right) = 0.015 \]
\[ \frac{c - 10}{1.5} = -2.17 \]
\[ c = 10 - 2.17(1.5) = 6.745 \]

3. The life span of oil-drilling bits depends on the types of rock and soil that the drill encounters, but it is estimated that the mean length of life is 75 hours. Suppose an oil exploration company purchases drill bits that have a life span than is approximately normally distributed with a mean equal to 75 hours and a standard deviation equal to 12 hours.

A. What proportion of the drill bits fail before 60 hours of use?

\[ P(X < 60) = P\left( Z < \frac{60 - 75}{12} \right) = P(Z < -1.25) = 0.1056 \]

B. What proportion of the drill bits fail between 62 and 78 hrs

\[ P(62 < X < 78) = P\left( \frac{62 - 75}{12} < Z < \frac{78 - 75}{12} \right) \]
\[ = P(-1.08 < Z < .25) = .5987 - .1401 = .4586 \]

C. At least how long do the best 5% of the drill bits last?

\[ P(X > c) = 0.05 \]
\[ P\left( Z > \frac{c - 75}{12} \right) = 0.05 \]
\[ \frac{c - 75}{12} = 1.645 \]
\[ c = 75 + 1.645(12) = 94.74 \text{hrs} \]