Since its introduction as a powerful graph-based method for image segmentation, the Normalized Cuts (NCuts) algorithm has been generalized to incorporate expert knowledge about how certain pixels or regions should be grouped. Prior works have incorporated expert knowledge in the form of hard constraints. We propose incorporating soft constraints by adding a penalty term to the cut cost:

\[
\gamma \rightarrow 0 \quad \text{soft constraints}
\]

\[
\gamma \rightarrow \infty \quad \text{hard constraints}
\]

As \(\gamma \rightarrow 0\) the solution approaches that of unconstrained NCuts while as \(\gamma \rightarrow \infty\) the solution approaches that of hard constrained NCuts. We can see that the best solution occurs when \(\gamma\) is somewhere in the middle, showing that soft constraints may provide an optimal solution.

**Incorporating Soft Constraints**

We propose incorporating soft constraints by adding a penalty term to the cut cost:

\[
P_y = \gamma \sum_{\forall y \in \mathcal{Y}} u_{y}^2\theta(y, \gamma)
\]

\[
P_f = \gamma \sum_{\forall f \in \mathcal{F}} u_{f}^2\theta(f, \gamma)
\]

where \(\gamma\) is the weighting parameter. When \(\gamma \rightarrow 0\) the solution approaches that of unconstrained NCuts while as \(\gamma \rightarrow \infty\) the solution approaches that of hard constrained NCuts. We can see that the best solution occurs when \(\gamma\) is somewhere in the middle, showing that soft constraints may provide an optimal solution.

We propose incorporating soft constraints by adding a penalty term to the cut cost:

\[
\gamma \sum_{\forall y \in \mathcal{Y}} u_{y}^2\theta(y, \gamma)
\]

**Choosing Weights**

Constraint weights \((\gamma, \beta)\) can have unpredictable effects when the number of constraints changes, so we suggest the following normalization strategy:

- Set equal constraint weights so that \(\Gamma = \mathbf{1}\) and \(F = \mathbf{1}\).
- Rescale \(U\) and \(B\) so that \(U^T U = (U^T U) = \text{tr}(D)\).

This normalization gives every constraint an equal amount of influence when \(\gamma = \beta\) and enables similar choices of \(\gamma\) and \(\beta\) across images.

**Conclusions**

Normalized Cuts can be easily extended to handle soft must-link and cannot-link constraints. The constraints can be provided by an expert user, and their soft nature allows their relative influence to be varied. Through experiments on real-world imagery, we show that the soft character of the constraints can achieve more robust results than Normalized Cuts with hard constraints.

**References**

