ACCOUNTING FOR CHANGING OVERLAP IN VARIATIONAL IMAGE REGISTRATION

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ABSTRACT

Any similarity measure used for image registration depends in some way on the region $\Omega$ describing the overlap between the floating and reference images. In variational registration, where the Gâteaux derivative of the similarity measure drives the registration, most literature implicitly assumes that $\Omega$ remains constant. This assumption is valid if homogeneous Dirichlet or sliding boundary conditions are chosen for the displacement field; however, it is invalid if any other type of boundary conditions are chosen, or if the similarity measure is computed over some masked portion of the overlap region. This article illustrates how these more general situations of different boundary conditions and/or masked regions can be accommodated in variational registration by explicitly accounting for the varying $\Omega$ in the Gâteaux derivative of the similarity measure.

Index Terms— Nonrigid registration, variational methods

1. INTRODUCTION

In any application requiring image registration, the similarity measure is computed from data drawn from the overlap region $\Omega$, which is defined as the intersection of the valid data regions in the floating and reference images. Frequently, valid data exists only in portions of the floating or reference images. Consider, for example, the axial slices from serial 3D chest CT scans of the same patient shown in Fig.'s 1(a) and 1(b). The particular reconstruction algorithm used to generate these CT scans only reconstructs data within a cylinder.

In other situations where nonrigid registration is employed, the choice of boundary conditions for the displacement field can impact the evolution of $\Omega$, even if valid data exists initially in the entire floating and reference images. If Neumann boundary conditions are chosen, for example, then the boundaries of $\Omega$ can change over the course of registration. In fact, the only boundary conditions typically used in registration that guarantee the constancy of $\Omega$ are homogeneous Dirichlet and sliding boundary conditions [1, 2].

One way to handle both of these situations is to construct masks that indicate valid sampling locations and track the masks over the course of registration. Figures 1(c) and 1(d) illustrate these masks for the axial slices of the 3D chest CT scans. As registration progresses, the masks deform; as the masks deform, their region of intersection ($\Omega$) deforms as well.

In these situations, then, a straightforward approach to image registration involves computing the image similarity measure by sampling the floating and references at points in $\Omega$. (See [3] for an example of such an approach.) However, if the computation of the deformation gradient does not appropriately account for the changing $\Omega$, problems can emerge near the boundary of $\Omega$.

At least two examples exist in the research literature of appropriately handling overlap regions in the deformation gradient. Schumacher et al. [4] and Ens et al. [5] illustrate how to compute Gâteaux derivatives of the sum of squared differ-
ence (SSD) and mutual information (MI) similarity measures with respect to the displacement field in the presence of additive masks. Both of these examples assume additive masks for the floating and reference images, and they gloss over the problem of how to deal with nondifferentiable region edges.

In this paper, we propose a technique that relies on multiplicative masks for accommodating changing overlap regions. The use of multiplicative masks naturally models situations such as that in Fig. 1 where the floating and reference images contain different regions of valid data. We show how the Chan-Vese trick [6] of regularizing the Heaviside function enables dealing with nondifferentiable edges. Finally, we present the new forms of the Gâteaux derivative of the mean squared difference (MSD) similarity measure for use in variational registration, and we illustrate how this derivative differs from the standard form that assumes constancy of \( \Omega \).

2. MATHEMATICAL PRELIMINARIES

Consider two images, a reference image \( R \) and a floating image \( F \), both as functions in \( \mathbb{R}^n \). Define a deformation \( \Phi : \mathbb{R}^n \rightarrow \mathbb{R}^n \) by \( \Phi(x) = x - u(x) \), and call \( u \) the displacement. The general form of the registration problem is given by:

\[
\min_u J(R, F, u) := J(R, F^u) + \alpha S(u),
\]

where \( J \) is a similarity measure that quantifies the dissimilarity between the reference image \( R \) and the deformed floating image \( F^u := F(\Phi) \), \( S \) is a regularizer that ensures that the minimization problem is well-posed and that the solution is smooth in some sense, and \( \alpha \) is a weighting parameter.

Necessary conditions for a minimum of \( J(R, F, u) \) are given by the Euler-Lagrange equations:

\[
\mathcal{A}(u(x)) = \alpha f(x; R, F^u),
\]

with suitable boundary conditions. The partial differential operator \( \mathcal{A} \) and force vector \( f \) arise from the Gâteaux derivatives of the similarity measure and regularizer, respectively. In this article, we assume that \( \mathcal{A} \) is the negative Laplacian operator, but we note that other operators [1, 2, 7] can be employed depending on the type of regularization desired.

We focus here on the mean squared differences (MSD) similarity measure commonly used in unimodal registration. The MSD is defined by:

\[
J_{\text{MSD}}(R, F^u) := \frac{1}{|\Omega|} \int_{\Omega} (R(x) - F^u(x))^2 \, dx.
\]

The corresponding Gâteaux derivative (under the assumption that \( \Omega \) is constant) is given by:

\[
dJ_{\text{MSD}}(R, F^u; w) = \int_{\Omega} \langle f_{\text{MSD}}(x; R, F^u), w(x) \rangle \, dx.
\]

where

\[
f_{\text{MSD}}(x; R, F^u) = \frac{2}{|\Omega|} (R(x) - F^u(x)) \nabla F^u(x),
\]

and where \( \langle \cdot, \cdot \rangle \) indicates the inner product.

3. DERIVATIVE OF REGION CONTENT

For either the situation where masks exist indicating valid data, or the situation where flexible boundary conditions are desired, we can describe the overlap region conceptually by Fig. 2. Consider the reference image to be defined over the region \( \Omega_R \subseteq \mathbb{R}^n \), and consider the deformed floating image to be defined over the region \( \Omega_{F^u} \subseteq \mathbb{R}^n \). The overlap region \( \Omega \) over which both images are defined is given by the intersection \( \Omega_R \cap \Omega_{F^u} \). (The situation where no masks exist and homogeneous Dirichlet or sliding boundary conditions are chosen is still covered; this case simply yields \( \Omega = \Omega_R = \Omega_{F^u} \).)

![Fig. 2. Regions over which reference image (left) and floating image (middle) are defined; overlap region \( \Omega \) as the intersection of these regions (right).](image)

Now, the content \( C(u) := |\Omega| \) of the overlap region is given by:

\[
C(u) = \int_{\mathbb{R}^n} H(\Theta_R(x)) \cdot H(\Theta_{F^u}(x)) \, dx,
\]

where \( \Theta_A(x) : \mathbb{R}^n \rightarrow \mathbb{R} \) is a function exhibiting the following properties:

\[
\Theta_A(x) > 0, \quad x \in \Omega_A, \quad \Theta_A(x) = 0, \quad x \in \partial \Omega_A, \quad \Theta_A(x) < 0, \quad \text{otherwise},
\]

and where \( H(\Theta) \) is the Heaviside step function, defined by:

\[
H(\Theta) = \begin{cases} 
1, & \Theta \geq 0 \\
0, & \Theta < 0, 
\end{cases}
\]

A natural choice for \( \Theta \) is the signed Euclidean distance transform [8], which can be computed in linear time.

If we approximate \( H \) in (6) with the regularized Heaviside function \( H_r \) defined in [6] by:

\[
H_r(\Theta) = \frac{1}{2} \left( 1 + \frac{2}{\pi} \arctan \left( \frac{\Theta}{\epsilon} \right) \right),
\]
we find that the Gâteaux derivative $dC(u; w)$ is given by:

$$
dC(u; w) = \int_{R^n} \langle f_C(x; R, F^u), w(x) \rangle \, dx, \quad (10)
$$

where

$$
f_C(x; R, F^u) \approx -H_\epsilon(\Theta_R(x)) \cdot H'_\epsilon(\Theta_{F^u}(x)) \cdot \nabla \Theta_{F^u}(x),
$$

and where

$$
H'_\epsilon(\Theta) = \frac{\epsilon}{\pi(\Theta^2 + \epsilon^2)}, \quad (12)
$$

is a regularized version of the Dirac delta function. Note that the choice $\epsilon = \pi^{-1}$ yields $H'_{\pi-1}(0) = 1$.

To get a visual sense of the Gâteaux derivative of the content of the overlap region, we observe in Fig. 3 the various components that are used to form the vector field $f_C(x; R, F^u)$ for the regions shown in Fig. 2. Figures 3(a)–3(b) show the regions $\Omega_R$ and $\Omega_{F^u}$ from Fig. 2 as binary masks, and Fig’s 3(c)–3(d) show the corresponding signed Euclidean distance transforms of these regions. Figures 3(e) and 3(f) show the regularized Heaviside function for the reference region and the derivative of the regularized Heaviside function for the floating region, respectively, assuming that the images are 300 pixels $\times$ 400 pixels with isotropic pixel spacing of 0.1 units, and that $\epsilon = \pi^{-1}$. Figure 3(e) shows the product of $H_\epsilon(\Theta_R(x))$ and $H_\epsilon(\Theta_{F^u}(x))$, and Fig. 3(h) illustrates the resulting vector field $f_C(x; R, F^u)$ overayed on the mask of the floating region. As can be seen in Fig. 3(h), the vector field $f_C(x; R, F^u)$ has nontrivial vectors only near the edges of $\Omega_{F^u}$ that are inside $\Omega_R$, and these vectors point outward from $\Omega_{F^u}$.

4. REVISITING THE DERIVATIVE OF MSD

In order to explicitly account for the regions $\Omega_R$ and $\Omega_{F^u}$ over which the reference and floating images are defined, we rewrite the MSD similarity measure (3) as:

$$
\mathcal{J}_{\text{MSD}}(R, F^u) := \frac{M(R, F^u)}{C(u)}, \quad (13)
$$

with

$$
M(R, F^u) := \int_{R^n} H(\Theta_R(x)) \cdot H(\Theta_{F^u}(x)) \cdot (R(x) - F^u(x))^2 \, dx
$$

and $C(u)$ defined as in (6). Using calculus of variations, we see that the revised Gâteaux derivative of $\mathcal{J}_{\text{MSD}}$ is given by

$$
d\mathcal{J}_{\text{MSD}}(R, F^u; w) = \int_{\Omega} \left\langle \hat{f}_{\text{MSD}}(x; R, F^u), w(x) \right\rangle \, dx,
$$

where

$$
\hat{f}_{\text{MSD}}(x; R, F^u) \approx f_C(x; R, F^u) \cdot \left( \frac{(F^u(x) - R(x))^2 - \mathcal{J}_{\text{MSD}}(R, F^u)}{|\Omega|} \right) + f_{\text{MSD}}(x; R, F^u) \cdot H_\epsilon(\Theta_R(x)) \cdot H_\epsilon(\Theta_{F^u}(x)).
$$

The second term in (16) corresponds to the restriction of the originally computed force vector (5) to the region $\Omega$. The first term in (16) is zero everywhere except on the boundary of $\Omega$ and corresponds to the correction that is necessary to accommodate the changing overlap region. If the correction term $f_C$ is omitted from (16), then registration performed using the resulting force vector is essentially equivalent to previous ap-
proaches (such as in [3]) that simply zero the force vector outside the mask.

5. EXAMPLE

To compare the effect of including or not including the correction term $f_C$ in the force vector, we illustrate what happens when we align the chest CT slices shown in Fig. 1. Figures 4(a) and 4(b) show the prior CT slice deformed into the frame of reference of the current CT slice according to deformation fields found from fluid registration [9] without and with the inclusion of the correction term $f_C$, respectively.

Figures 4(c) and 4(d) show the differences between the current CT slice and the aligned prior slices. A comparison of these figures shows that when the correction factor $f_C$ is not included, the resulting fluid registration stretches the prior slice near the mask boundaries, driving it outward. This is verified by observing the magnitude of the resulting displacement fields, shown in Fig.’s 4(e) and 4(f) (with black/white indicating 0cm/1.5cm magnitudes, respectively). We see that unless the correction factor $f_C$ is included, the regions at the boundary of the mask yield the largest deformation, even though this is not physically accurate.

6. CONCLUSION

In this article, we have shown how variational registration techniques can be designed to appropriately handle changing overlap regions by incorporating a correction factor in the force vector field. As illustrated by the CT slice registration example, this correction factor ensures that the resulting deformations do not yield implausible results near the region boundary.

7. REFERENCES


Fig. 4. Aligning CT slices from Fig. 1 using the force vector defined in (16) without and with the correction term $f_C$. information,” in Proc. SPIE Medical Imaging: Image Processing, 2007, vol. 6512.


