Quiz 3

Please show all work. Partial credit may be given, but only if all work is shown.

1. Find the domain of the vector function:
   \[ \mathbf{r}(t) = \langle t^2, \sqrt{t-1}, \sqrt{5-t} \rangle \]

   The first component is defined for \(-\infty < t < \infty\).
   The second component is defined for \(t \geq 1\).
   The third component is defined for \(t \leq 5\).

   Thus, the domain for \(\mathbf{r}(t)\) is \(1 \leq t \leq 5\).

2. Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point:
   \[ x = t, \quad y = \sqrt{2} \cos t, \quad z = \sqrt{2} \sin t \]
   at \( (\pi/4, 1, 1) \)

   \[ \mathbf{r}'(t) = \langle 1, -\sqrt{2} \sin(t), \sqrt{2} \cos(t) \rangle \]

   \[ \mathbf{r}'(\pi/4) = \langle 1, -1, 1 \rangle \]

   \[ x = \pi/4 + t, \quad y = 1 - t, \quad z = 1 + t \]

3. The curves \(\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle\) and \(\mathbf{r}_2(t) = \langle \sin t, \sin 2t, t \rangle\) intersect at the origin. Find their angle of intersection correct to the nearest degree. Note: The angle of intersection of two curves is the angle between their tangent lines at the point of intersection.

   \[ \mathbf{r}_1'(0) = \langle 1, 0, 0 \rangle \quad \text{and} \quad \mathbf{r}_2'(0) = \langle 1, 2, 1 \rangle \]

   Thus, the cosine of the angle between the tangent vectors is
   \[ \cos \theta = \frac{\langle 1,0,0 \rangle \cdot \langle 1,2,1 \rangle}{\sqrt{1+0+0} \sqrt{1+4+1}} = \frac{1}{\sqrt{6}} \]

   and
   \[ \theta = \cos^{-1} \left( \frac{1}{\sqrt{6}} \right) \approx 66^\circ \]