Top and side surfaces of a cubical furnace are black, and are maintained at uniform temperatures. Net radiation heat transfer rate to the base from the top and side surfaces are to be determined.

**Assumptions**
1. Steady operating conditions exist
2. The surfaces are opaque, diffuse, and gray.
3. Convection heat transfer is not considered.

**Properties**
The emissivities are given to be $\varepsilon = 0.7$ for the bottom surface and 1 for other surfaces.

**Analysis**
We consider the base surface to be surface 1, the top surface to be surface 2 and the side surfaces to be surface 3. The cubical furnace can be considered to be three-surface enclosure with a radiation network shown in the figure. The areas and blackbody emissive powers of surfaces are

$$A_1 = A_2 = (10 \text{ ft})^2 = 100 \text{ ft}^2 \quad A_3 = 4(10 \text{ ft})^2 = 400 \text{ ft}^2$$

$$E_{b1} = \sigma T_1^4 = (0.1714 \times 10^{-8} \text{ Btu/h ft}^2 \cdot \text{R}^4)(800 \text{ R})^4 = 702 \text{ Btu/h ft}^2$$

$$E_{b2} = \sigma T_2^4 = (0.1714 \times 10^{-8} \text{ Btu/h ft}^2 \cdot \text{R}^4)(1600 \text{ R})^4 = 11,233 \text{ Btu/h ft}^2$$

$$E_{b3} = \sigma T_3^4 = (0.1714 \times 10^{-8} \text{ Btu/h ft}^2 \cdot \text{R}^4)(2400 \text{ R})^4 = 56,866 \text{ Btu/h ft}^2$$

The view factor from the base to the top surface of the cube is $F_{12} = 0.2$. From the summation rule, the view factor from the base or top to the side surfaces is $F_{11} + F_{12} + F_{13} = 1 \rightarrow F_{13} = 1 - F_{12} = 1 - 0.2 = 0.8$

since the base surface is flat and thus $F_{11} = 0$. Then the radiation resistances become

$$R_1 = \frac{1 - \varepsilon_1}{A_1 \varepsilon_1} = \frac{1 - 0.7}{100 \text{ ft}^2 (0.7)} = 0.0043 \text{ ft}^{-2} \quad R_{12} = \frac{1}{A_1 F_{12}} = \frac{1}{100 \text{ ft}^2 (0.2)} = 0.0500 \text{ ft}^{-2}$$

$$R_{13} = \frac{1}{A_1 F_{13}} = \frac{1}{100 \text{ ft}^2 (0.8)} = 0.0125 \text{ ft}^{-2}$$

Note that the side and the top surfaces are black, and thus their radiosities are equal to their emissive powers. The radiosity of the base surface is determined

$$\frac{E_{b1} - J_1}{R_1} + \frac{E_{b2} - J_1}{R_{12}} + \frac{E_{b3} - J_1}{R_{13}} = 0$$

Substituting,

$$\frac{702 - J_1}{0.0043} + \frac{11,233 - J_1}{0.500} + \frac{56,866 - J_1}{0.0125} = 0 \rightarrow J_1 = 15,054 \text{ W/m}^2$$

(a) The net rate of radiation heat transfer between the base and the side surfaces is

$$Q_{31} = \frac{E_{b3} - J_1}{R_{13}} = \frac{(56,866 - 15,054) \text{ Btu/h ft}^2}{0.0125 \text{ ft}^{-2}} = 3.345 \times 10^6 \text{ Btu/h}$$

(b) The net rate of radiation heat transfer between the base and the top surfaces is

$$Q_{12} = \frac{J_1 - E_{b2}}{R_{12}} = \frac{(15,054 - 11,233) \text{ Btu/h ft}^2}{0.05 \text{ ft}^{-2}} = 7.642 \times 10^4 \text{ Btu/h}$$

The net rate of radiation heat transfer to the base surface is finally determined from

$$Q_1 = Q_{21} + Q_{31} = -76,420 + 3,344,960 = 3.269 \times 10^6 \text{ Btu/h}$$

**Discussion**
The same result can be found form

$$\frac{J_1 - E_{b1}}{R_1} = \frac{(15,054 - 702) \text{ Btu/h ft}^2}{0.0043 \text{ ft}^{-2}} = 3.338 \times 10^6 \text{ Btu/h}$$

The small difference is due to round-off error.