Experiment 6

Bernoulli Equation
and Flow Meters

Purpose

The objective of this experiment is to experimentally verify the Bernoulli Equation and to show how pressure measurements can be used to measure flow rates in pipes.

Apparatus

This laboratory exercise involves two separate pieces of equipment. One setup is used to verify the Bernoulli equation and the other is used to measure flow rate of water with two different types of obstruction meters.

Figure 6.1 depicts the apparatus used to verify the Bernoulli equation. It consists of a blower connected to a duct with a Venturi. At the exit of the duct is a support for a Pitot probe. The support allows the probe to be positioned at different axial locations in the duct.

The Pitot tube (see Munson et al., §3.5) is a device that enables simultaneous measurement of the total and static pressure of a moving fluid. For the Pitot probe used in this experiment the distance between the total and static pressure ports is 11/16 inch. This dimension will be important while making pressure measurements in regions of the duct with varying cross-sectional area. On the opposite end of the Pitot probe from the measuring tip are taps that allow the static and total pressure lines to be connected to a pressure measuring device. Not shown in Figure 6.1 are three U-tube manometers that are used to measure the static pressure, dynamic pressure and total pressure from the Pitot probe.

Figure 6.2 is a schematic of the apparatus used to demonstrate obstruction-type flow meters. The apparatus is a flow loop with a Venturi meter, a sharp-edged orifice, a paddle-type flow meter, a pump, and a collection tank. The pressure taps around the flow loop have quick-disconnect connectors. By switching taps it is possible to measure pressure differentials at different points around the loop with a single U-tube manometer.
Figure 6.1: Apparatus for verifying the Bernoulli Equation.

Figure 6.2: Flow loop for testing obstruction-type flow meters.


Theory

Bernoulli Equation

The Bernoulli equation applies to steady, incompressible flow along a streamline with no heat or work interaction. One form of the Bernoulli equation is

\[
\frac{p_0}{\rho} = \frac{p_1}{\rho} + \frac{1}{2} \rho V_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2} \rho V_2^2 + gz_2
\]

where \( p \) is the pressure, \( \rho \) is the density, \( V \) is the fluid velocity, \( g \) is the acceleration of gravity, and \( z \) is the elevation measured from an arbitrary datum. The subscripts 1 and 2 denote two positions along the streamline. The total or stagnation pressure, \( p_0 \), is a measure of total energy of the flowing stream. When the Bernoulli equation applies the stagnation pressure is constant along a streamline.

Figure 6.3 is a sketch of the flow field near the tip of the Pitot probe. The streamline that terminates at point \( A \) is called the stagnation streamline because as the fluid approaches \( A \) it decelerates until it has zero velocity. The fluid velocity is \( V \) on the stagnation streamline far upstream of \( A \). If we assume that the deceleration is reversible, the pressure tap at point \( A \) measures the total pressure in the vicinity of the tip. Note that there is no flow through the Pitot tube.

The Pitot probe is a small aerodynamic body that does not significantly disturb the flow field except for the stagnation streamline. Although the streamlines curve as the fluid passes around the tip there is negligible change in velocity for fluid that follow streamlines near the probe. Along the streamline through point \( C \), for example, the fluid velocity is assumed to be constant. In addition, over distances on the order of the Pitot tube diameter, elevation changes are negligible. All the streamlines in Figure 6.3, therefore, have the same stagnation pressure

\[ p_{0,A} = p_{0,C} \]

There is a different relationship between points \( B \) and \( C \). Since there is no velocity in the direction between \( B \) and \( C \) the static pressure is the same at these points,

\[ p_B = p_C \]

In other words the pressure tap at point \( B \) measures the static pressure in the vicinity of the probe tip. Combining the foregoing equations and assumptions, the pressure difference between point \( A \) and point \( C \) is the dynamic pressure

\[ p_A - p_B = \frac{1}{2} \rho V^2 \]

Obstruction Flow Meters

Obstruction type flow meters work on the principle that changes in flow area result in changes in fluid velocity as required by continuity, and the changes in velocity result in changes in fluid pressure. The theory of flow meters is discussed in § 3.6.3 and § 8.6 of Munson et al. The volumetric flow rate through an obstruction type flow meter is

\[ Q = CA \sqrt{\frac{\Delta p}{\rho (1 - \beta^4)}} \]

where \( \beta = d/D \), \( d \) is the diameter of the orifice (or throat of the Venturi), and \( D \) is the diameter of the pipe.
Procedure

Verification of the Bernoulli Equation

By sliding the Pitot tube through its support, traverse the flow channel along the axis and record the static, dynamic (velocity) and total pressures indicated by the U-tube manometers. In sections of the duct where the flow area is changing the Pitot tube must be moved 11/16 inch between the total and static pressure measurements to insure the pressure readings are made at locations with the same cross-sectional area.

Obstruction Flow Meter Experiment

1. Turn on the pump and choose a flow rate.

2. Move the pressure lines connected to the U-tube manometer to the taps on either side of one of the flow meter (orifice or Venturi).

3. Record the pressure drop across the flow meter (orifice or Venturi).

4. Repeat steps 2 and 3 for a total of six different flow rates.

5. Repeat steps 2 through 4 for the other flow meter

Report

Verification of the Bernoulli Equation

1. Plot the total pressure along the duct.

2. Plot the variation of the centerline velocity along the duct.

3. Calculate the flow rate through the duct.

4. When using a Pitot tube in a duct of constant cross-sectional area the total and static pressure are measured at two streamwise locations. What assumption is being made when the distance between these locations is neglected?
Obstruction Flow Meter Experiment

1. Calculate the discharge coefficient, $C$, for each flow meter at each flow rate.

2. Calculate the Reynolds number for each flow rate.

3. Make a plot of $C$ versus $Re$ on semi-log paper for the orifice and the Venturi. Discuss any discrepancies with your results and the curves in Munson et al.

Reference