The Needle in the Haystack: the physics of prolate granular piles

Scott Franklin
svfsp@rit.edu
http://piggy.rit.edu/franklin

Undergraduate Researchers
Kevin Stokely
Ari Diacou
Jesus Benhumea
Ken Desmond
Saul Lapidus
Peter Gee

PRE May, 2003
Petroleum Research Fund
Granular Basics

- Granular materials: collections of macroscopic (\(\sim\) mm) particles interacting through contact forces (e.g. friction)

- Thermal energy irrelevant (\(k_B T \sim 10^{-21} \text{ J} \ll PE \sim 10^{-5} \text{ J}\))
  - systems “frozen” in metastable state, unable to move to lower energy state without external help

- exhibit solid, liquid, gas-like behavior

- “density” of a granular material is not well defined (packing fraction: percentage of space occupied)
  - 3-d spheres: 52\% < \(\rho\) < 74\%
  - 2-d disks: 78\% < \(\rho\) < 91\%
  - long, thin rods in 3-d: 5\% < \(\rho\) < 91\%
  - long, thin rods in 2-d: ?? < \(\rho\) < 100\%
Why Extremely Prolate Granular Materials?

- Large *aspect ratio* ($\alpha = L/D \gg 1$), rods become entangled
  - far more rigid than ordinary sand, maintain shape of initial container

- Ability to move is anisotropic (direction dependent)
  - move easily along axis, rotations or transverse motions quite difficult
Basis for Interest: A Bucket of Nails

- Why do pickup sticks form such rigid piles?

- 1” steel ball pulled through nails ($L = 3.3\text{cm}, L/D = 20$), (Vivian Garcia-Arnold, Dickinson, 1999)
Irregular Force Fluctuations

- motion is stick-slip, (linearly increasing force, indicating ball at rest)

- fluctuation spectrum goes as $1/f$, (ordinary sand $\sim 1/f^2$)
Logarithmic Settling of Sand

- Knight et al. (PRE 51, 1995) found settling even after $10^4$ taps

- $\alpha = 3.9$ rods compacted when tapped
- rods aligned with sidewalls (Villarruel, 2000)
- cylinder <3 rod lengths across
Packing and Force Distribution

Ordered packing yields uniform force distribution

Slight disorder produces localized force chains (Parke Rhoads, 1998)

- large force chains more common than Gaussian
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\[
\begin{align*}
F^y_{\text{tube} \rightarrow \text{sand}} &= W \\
F^y_{\text{tube} \rightarrow \text{sand}} &= \mu F^x_{\text{sand} \rightarrow \text{tube}} \\
\mu &\approx 0.2 \\
\implies F^x_{\text{sand} \rightarrow \text{tube}} &\approx 5W
\end{align*}
\]
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\[ F_{\text{tube} \rightarrow \text{sand}}^y = W \]
\[ F_{\text{tube} \rightarrow \text{sand}}^y = \mu F_{\text{sand} \rightarrow \text{tube}}^x \]
\[ \mu \approx 0.2 \]
\[ \Rightarrow F_{\text{sand} \rightarrow \text{tube}}^x \approx 5W \]

\[ 2F \sin(\theta) = W \]
\[ \Rightarrow F = \frac{W}{2 \sin(\theta)} \]
Who cares?

- $\sim 40\%$ loss in production from factories from handling granular materials

- great industrial significance
  - petroleum: extract oil through (granular) soil
  - chemical: maximize reaction rate between liquid and particulate reactants
  - pharmaceutical: uniformly mix particulate constituents for medicine

- large percentage of GDP estimated to involve granular materials (33%???)

- Physicists: appropriate theoretical framework for something both solid and liquid?
Static Piles of Rods: Experiment

- \( \sim 2000 \) acrylic rods \( (\alpha = 12) \) sandwiched between two plates
  - rod diameter: 1/16”  rod length: 3/4”  \( L/D = 12 \)
  - plate separation: 3/32”
  - digital picture analyzed for rod position and orientation

\[ L/D = 12 \]

- Packing fraction \( \phi = 68\% \pm 3\% \)
- Orientational order parameter

\[ Q \equiv \frac{1}{N} \sum_i \cos [2\theta_i] \in [-1, 1] \]

- particles tend towards horizontal: \( Q = 0.33 \pm 0.05 \)
Effect of Bottom Boundary

- bottom boundary’s effect quickly decays with height
- asymptote in simulations w/random bottom profile
Simulations

- Monte-carlo, not Molecular Dynamics
  - don’t consider Newton’s Laws, forces, etc.
  - approximate many collisions with random displacements

1. Place needles at random (discrete) lattice locations and orientations (box length=10$L$)
2. Choose random needle, attempt to move random displacement and rotation
3. Move needle incrementally from initial to final location
   - if needle intersects another along the way, immediately stop and leave it at last non-intersecting location
Comparison of Experimental and Simulated Piles

- Visual similarities between piles
Angular Distribution of Rods

- Simulations have more horizontal rods, consequences for packing
- Experiment has not insignificant number of vertical rods
- Friction encourages vertical rods, arches
Distribution of Voids

- Void defined as contiguous dark pixels bounded by rods
- $V(A)$: Number of voids with area $A$

voids of area $A$ occupy space $N(A)A \propto A^{-1.37}$
Consequences of $V(A) \propto A^{-2.37}$

- Total area occupied by voids of area $A$ is $V(A) \times A \propto A^{-1.37}$
  * smaller voids occupy more total area than larger voids

- Total area occupied by voids is $A_{\text{voids}} = \int V(A) \, A \, dA$

- $V(A) \sim A^{-eta = -2.37} \implies A_{\text{voids}} = \int A^{-\beta} \, A \, dA = \int A^{-1.37} \, dA$
  * integral converges, so pile area remains finite
  * mean void area exists

- $\langle A^2_{\text{void}} \rangle = \int A^{-\beta} \, A^2 \, dA = \int A^{-0.37} \, dA$, no mean square void area?
Particle Alignment

- Angular Correlation Function $Q(r)$
  * $Q(r) = \langle \cos (2\Delta \theta_{ij}) \rangle_{r_{i,j}=r}$, $0 < Q < 1$
  * rods very close together must be aligned, so $Q(r \to 0) \approx 1$

  ![Diagram of rods]

  * Rod $B$, closer to horizontal rod, has fewer available angles
  * rods separated by more than length $L$ can assume any relative angle, so $Q(r > L) \approx 0$

- Compare with random distribution where all angles equally probable
  * testing whether rods prefer to align with their neighbors
Angular Correlation

- Enhanced alignment for $r < L$ (particle-particle interactions)
- $P(\theta)$ for experiment, simulation not flat
Angular Correlation II

- $P(\theta)$ for experiment, simulation not flat
- $Q_\infty = \int P(\theta)P(\phi)\cos[2(\theta - \phi)]\,d\theta d\phi = 0.16\exp,\,0.53\,(\text{sim})$

![Graph showing data for varying aspect ratios.](image)

- Universal functional form for all aspect ratios?
Is $\alpha = 12$ long enough?

- agreement between simulation and experiment encourages extending simulation to even larger aspect ratios

- Simulated piles of longer rods show no more or less angular order

- $\alpha = 12$ just might be in asymptotic limit
Contact Number

• \( \langle c \rangle \): average number of *touching* neighbors

* Might think \( \langle c \rangle \) increases with \( \alpha \)

- Particle screening keeps \( \langle c \rangle \) constant for large \( \alpha \)

- Rods have *fewer* touching neighbors than round disks \( (\langle c \rangle \approx 5) \)
- Simple geometric model: \( N(L) = \frac{\pi \langle c \rangle}{L^2} \)
  - one particle per \( A_{\text{excl}} \): \( N(\alpha) = \frac{1}{A_{\text{excl}}(\alpha)} \)
  - If each rod touches \( \langle c \rangle \) neighbors, \( N(\alpha) = \frac{2 \langle c \rangle}{A_{\text{excl}}(\alpha)} \)
  - \( A_{\text{excl}}(L) = \frac{2}{\pi} L^2 + O(Lw) \) (Balberg, 1984)
Conclusions about static piles

- Formed statistically consistent 2-d piles of large aspect-ratio (anisometric) granular materials
  - packing fraction for $\alpha = 12$, $\phi = 68\% \pm 3\%$
  - particles tend towards horizontal ($Q \approx 0.3$)
  - significant angular correlation between neighbors
  - voids distributed as power law with exponent $-\beta = -2.37$
  - Monte-carlo results consistent with experiment

- simple geometric model captures scaling of simulation but under-predicts number density by factor of 2
What’s Next?

- **Angle of Repose**

  Drum filled with sand is slowly rotated. At critical angle avalanche starts. Avalanche stops at a lesser critical angle.

\[ \alpha = 10 \quad \alpha = 40 \]
Angle of Repose: $\alpha = 10$

- average over 5000 pictures (3 minutes of data)
Packing Fraction $\phi$

- Packing fraction fluctuates around reasonable value ($\phi(12) = 0.68 \pm 0.03$)

- $\phi(10) = 0.69 \pm 0.02$, $\phi(40) = 0.56 \pm 0.04$
Events

- How likely are large/small events?
Event distributions

\[ \alpha = 10 \]

\[ \alpha = 40 \]
Boundary length

- Longer aspect ratio has more jagged surface, longer "coastline" (4977/3521) but std. dev. the same (16%)

- Doesn’t appear to have a fractal dimension
Conclusions

- Rotating experiments beginning, searching for good way of analyzing data
  - packing fraction, voids distributions, avalanche distributions

- Forming Connected Networks in 2-d Granular Piles
  - increase volume fraction, aspect ratio independently
  - theory of jamming suggests these are similar, yet large aspect ratio piles qualitatively different than those at lower aspect ratios
  - push disks through horizontal network, Hypothesize: solid body movement of piles above critical aspect ratio

- Molecular dynamics simulations
  - Recent work (e.g. Silbert et al., PRE 051302 64, 2001) incorporated state-of-the-art contact functions to simulate collections of spheres
  - simulations keep track of elastic tangential displacement, elastic deformation caused by two particles in contact

- Long way to go, really don’t know much about needles