Jamming in 2d Prolate Granular Materials

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Background

- Philipse observed piles of long, thin rods to pour from a bucket in a single, solid-like plug

- Albert et al. (2000) measured fluctuations in force on an object moving through round particles to scale as $1/f^2$
  - 1” steel ball pulled through a bucket of $\alpha = 20$ nails showed stick slip motion with spectrum $\propto 1/f$

- To et al. (2002) found 2-d disks to set up arches of up to 9 particle diameters ($d$) spanning a hopper of width $w = 4.05d$, with jamming occurring about 25% of the time
Experimental Setup

- Particles: $L = 2.5''$, $w = 0.125''$, $\alpha = 20$, Channel: width $W = 4 - 7L$
- 3/4”wide metal rectangle pushed through the pile at 1-2 cm/s)
- record the force on the foot, videotape the pile
- Count how many particles are pushed off the table
Video Images

$N = 100 \ (\phi \approx 10\%)$

$N = 280 \ (\phi \approx 29\%)$
Video Analysis

- subtract successive frames to see which rods have moved

- As you might expect, as packing fraction increases so does extent of disturbance
Number of bright pixels in subtracted image

Time (seconds)
• Larger packing fractions have higher force (common sense)

• Better resolution or more uniform initial conditions needed before comparison can really be made
Preliminary Indications of Jamming Threshold

- Number of rods pushed of plate increases at around $\phi = 0.45$
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- Number of rods pushed off the plate increases at around $\phi = 0.45$

![Graph showing the relationship between packing fraction and the percentage of rods pushed off the table.]

- Large error bars due to few trials and non-uniform initial conditions.
Quantifying Initial Conditions

- Angular distribution alone doesn’t capture particle alignment

- Studies on static piles let to the idea of an angular correlation function $Q(r/L)$

- For an isotropic distribution, $Q_i(r/L)$ can be determined analytically as an integral over all allowable configurations (excluded area becomes important)
• Define order parameter \( O = \frac{\int |Q(r/L) - Q^i(r/L)| dr}{\int [1 - Q^i(r/L)] dr} \) which has the properties that

\[
O = \begin{cases} 
0 & \text{system is isotropic} \\
1 & \text{system is completely ordered}
\end{cases}
\]
Conclusions

- There may be a transition to solid-body like motion in 2-d prolate granular materials

- Quantifying the initial conditions will be crucial for achieving reproducible results

  - Use ideas developed in analyzing static piles