Homework 3 Due Thursday October 1

1. Suppose we determine that an object of mass $m$ has a velocity $v(x) = \alpha/x$, where $\alpha$ is a constant. If $x = 1$ at $t = 0$,

(a) Find the force as a function of position, $F(x)$.

(b) Find the position as a function of time, $x(t)$.

2. A block moves along a horizontal surface that has a heavy lubricating oil on it, so that the force is $F(v) = -cv^{3/2}$. Initially the block has a position $x = 0$ and velocity $v = v_0$. Find the distance the block moves before coming to rest.

3. A bullet is shot upwards from a gun with an initial velocity $v_0$ and position $x = 0$. Assume up is positive, and quadratic air drag.

(a) Show that while the bullet rises, using $k = c_2/m$, the speed varies as

$$v^2 = A \exp(-2kx) - \frac{g}{k}$$

and express the integration constant $A$ in terms of known constants and variables.

(b) Show that on falling the speed varies as follows, where $x = 0$ is the point where the bullet has zero velocity.

$$v^2 = \frac{g}{k} - B \exp(2kx)$$

and express the integration constant $B$ in terms of known constants and variables.

(c) It is easy to see that $v_{\text{terminal}} \equiv v_t = \sqrt{g/k}$. Show that the speed of the bullet when it returns to its starting point is

$$v^2 = \frac{v_0^2 v_t^2}{v_0^2 + v_t^2}$$

(d) If a ping-pong ball, mass 2.7 g and diameter 4.0 cm, is shot upwards in air with a speed 7.0 m/s, (i) what is the maximum height reached? (ii) what is its speed upon returning to its starting height?