Homework 1 Due Thursday Sept. 17

1. Consider a cube with sides of unit length. One corner coincides with the origin, (0,0,0), and the cube is in the first octant, i.e. the sides extend along positive axes. (If you take solid state physics you will do lots of this when discussing crystallography.)

   a. Find the vector that represents a major diagonal extending from the origin to the most distant corner. Express using unit vectors and give its magnitude, and write an expression for the unit vector along this direction.

   b. Find the vector that represents a major diagonal extending from the corner on the x-axis. Express using unit vectors.

   c. Find a vector that represents a face diagonal along the face lying in the x-y plane.

   d. Find the angle between the vectors in parts (a) and (c).

   e. Find the angle between the vectors in parts (b) and (c).

2. Consider the vector $3\hat{i} - 2\hat{j} + \hat{k}$. Find a unit vector representation using the triad $\hat{i}'\hat{j}'\hat{k}'$ of this vector in a new coordinate system that is rotated about the y-axis by an angle of 30°.

3. A small ball is fastened to an elastic band and whirled in an elliptical path so that

   \[ \vec{r}(t) = \hat{i}2b\sin\omega t + \hat{j}3b\sin\omega t \]

   where $b$ and $\omega$ are constants.

   (a) Find the speed of the ball as a function of time.

   (b) Find the speed when the ball is closest to the origin, $t = \pi/2\omega$ and farthest from the origin, $t = 0$.

4. A bee flies out from the hive in a spiral path with

   \[ r = be^{kt} \quad \theta = ct \]

   Show that the angle between the velocity and the acceleration is constant. $b$, $c$, and $k$ are constant.

5. An object moves in plane polar coordinates so that

   \[ r = bt \quad \theta = ct^2/b \]

   with $b$ and $c$ constant. Find the expressions for $\vec{v}$ and $\vec{a}$, and find the magnitudes of the velocity and acceleration.

6. Prove that $\frac{d}{dt}[\vec{r} \cdot (\vec{v} \times \vec{a})] = \vec{r} \cdot (\vec{v} \times \vec{a})$.

7. Prove that $\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$. Note the cyclic form of this.