Homework 5

Due Feb 3

1. Consider a box of mass \( m \) with edges of length \( 2a \) along the \( y \)- and \( z \)-axes, and length \( 3a \) along the \( x \)-axis.

(a) Find the moment of inertia tensor for the box using the \( x \)-, \( y \)-, \( z \)-axes and the axis origin.

(b) Find the principal moments and the unit vectors for the principal axes (I suggest using the software that I mentioned.)

(c) Suppose that the cube is rotated about the \( x \)-axis with angular speed \( \omega \). Find the angle between \( \vec{L} \) and \( \vec{\omega} \).

(d) Suppose that the cube is rotated about a body diagonal (i.e. from \( (0,0,0) \) to \( (3,2,2)a \)) with angular speed \( \omega \). Find the angle between \( \vec{L} \) and \( \vec{\omega} \).

(e) Suppose that the cube is rotated about the axis described in part (d) with angular speed \( \omega \). Find the scalar moment of inertia about this axis.

(f) For the tensors we must modify the parallel axis theorem. Suppose that the inertia tensor is \( \mathbf{I}_{cm} \) about an axis through the center of mass. For a parallel axis through another point \( O' \) located at \( \vec{d} = (d_1, d_2, d_3) \), the elements of \( \mathbf{J}_{O'} \) are

\[
J_{ij} = I_{ij} + m \left( d^2 \delta_{ij} - d_i d_j \right)
\]  

where \( d \) is the magnitude of \( \vec{d} \) and \( \delta_{ij} \) is the Kronecker delta: \( \delta_{ij} = 1 \) if \( i = j \), and \( \delta_{ij} = 0 \) if \( i \neq j \).
Use the inertia tensor around the corner to determine the inertia tensor about parallel axes through the center of mass.

(g) Suppose the box is rotated about the $x$-axis. Find the required torque components relative to the principal axes.

(h) Suppose the box is rotated about the $y$-axis. Find the required torque components relative to the principal axes.

2. Recall the sphere with a hollow sphere removed from it (HW 3). Make the origin at the center of the large sphere (NOT the center of mass.)

(a) What are the principal axes for this object?

(b) What are the principal moments for this object?

3. Consider a square laminar plate of mass $m$ and side $a$. The principal axes relative to the center of mass are the symmetry axis-3 normal to the plate, and axes-1 and 2 parallel to the sides of the plate. The plate is thrown in the air so that it rotates freely under zero torque.

a) Find the moment of inertia tensor relative to the principal axes.

b) The plate is rotated about an axis that makes an angle of 45° with the symmetry axis in the 1-3 plane. Find the angular momentum relative to the 123-axes.

c) What is the angle between the angular momentum and the 3-axis? Hint: Consider dot-products. Refer to Figure 9.6.4 and decide what angle on that diagram this represents.

d) What is the angle between the angular momentum and $\vec{\omega}$? Hint: Again consider dot-products. Refer to Figure 9.6.4 and decide what angle on that diagram this represents.

e) What is the rate of precession of the plate about the invariable line ($\vec{L}$)?

f) What is the wobble rate, the rate of precession of symmetry axis about the invariable line, $\dot{\phi}$?