Vectors and Basic operations

> A vector is an ordered pair or arrow, denoted \( \vec{A} = (A_1, A_2, A_3) \).

Here \( A_1, A_2, A_3 \) are the \( x-, y-, \) and \( z-\) components of the vector in a cartesian coordinate grid.

> The tail of the arrow is placed at the origin, and the head at the point \((A_1, A_2, A_3)\).

All vectors parallel to \( \vec{A} \) and of the same length as \( \vec{A} \) are equivalent to \( \vec{A} \).

(Think "raining arrows")

This is called the AFFINE property of vectors.

> **Adding** \( \vec{A} = (A_1, A_2, A_3) \) \( \vec{B} = (B_1, B_2, B_3) \)

\( \vec{A} + \vec{B} = (A_1 + B_1, A_2 + B_2, A_3 + B_3) \)

> **Scalar multiplication**: If \( \lambda \) is any real number, then \( \lambda \vec{A} = (\lambda A_1, \lambda A_2, \lambda A_3) \).

In particular, if \( \lambda = -1 \), we have the negative, or opposite vector, \( -\vec{A} = (-A_1, -A_2, -A_3) \).
> **SUBTRACTING**: $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

> **Length or NORM**: $\|\vec{A}\| = \sqrt{A_1^2 + A_2^2 + A_3^2}$

> **PRODUCT**: The product of two vectors can be taken in a number of ways—we'll deal with two, the dot and cross product.

(i) **DOT**: $\vec{A} \cdot \vec{B}$ is a real number defined as

1. $\vec{A} \cdot \vec{B} = A_1B_1 + A_2B_2 + A_3B_3$

   If $\theta$ is the angle between $\vec{A}$ and $\vec{B}$, then this is also the same as

2. $\vec{A} \cdot \vec{B} = \|\vec{A}\| \|\vec{B}\| \cos \theta$

   By using (1) and (2) together, the angle between two vectors in 3-D space can be determined.

(ii) **CROSS**: $\vec{A} \times \vec{B}$ is a vector $\vec{C}$ that's perpendicular to both $\vec{A}$ and $\vec{B}$. $\vec{C}$ defines the 'other' dimension.

   $\|\vec{C}\| = \|\vec{A}\| \|\vec{B}\| \sin \theta$

   The direction of $\vec{C}$ is given by the RIGHT HAND rule: Bring $\vec{A}$ toward $\vec{B}$, then thumb points along $\vec{C}$.

   Also, $\vec{C} = (\{A_2B_3 - A_3B_2\}, \{A_3B_1 - A_1B_3\}, \{A_1B_2 - A_2B_1\})$. #