A NEW MODEL FOR THE SUCTION PRESSURE UNDER A CONTACT LENS

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We present a new approach to compute the suction pressure under a soft contact lens. When a soft contact lens is placed on an eye, it is subjected to forces from the tear film in which it is immersed and from the blinking eyelid. In response, the lens bends and stretches. The equilibration of these forces generates a suction pressure that keeps the lens on the cornea. In this paper, we develop a mathematical model of the elastic equilibrium of a soft contact lens that allows us to predict the suction pressure distribution under such a lens. We explore the influence of the shape of the lens and its elastic properties on the suction pressure.

Keywords: Contact Lens; Suction Pressure; Elastic Tension.

1. Introduction

The US contact lens industry is a multi-billion dollar business; 34 million Americans wear contact lenses. Ophthalmologists report that the main reasons for individuals stop wearing contact lenses are dryness and other sources of discomfort.1 Dropout rates for various countries range from 15% to 30%.2 Comfort is essential to successful product design. The current state-of-the-art contact lens design process is primarily empirical. A contact lens is manufactured; a clinical trial is run in which measurements are collected; and then the lens design is adjusted. We hope to use mathematical modeling to improve the design process.

We consider a soft hydrogel contact lens that conforms to the cornea and the sclera (the white part of the eye). The contact lens is separated from the eye surface by a 2–3 micron thick tear film called the post-lens tear film.3 When a lens is first placed on eye, the forces from the blink stretch the contact lens, pushing tear fluid out at the edges, and conforming the lens to the eye. Once the eye opens, the lens slowly relaxes back toward its original shape. The viscous drag of the post-lens tear film resists this relaxation, and this induces a pressure distribution, the suction pressure, under the lens. By design, the relaxation time of the contact lens

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is longer than a typical time of 4–5 s between blinks. After insertion, the contact lens is completely immersed in tear fluid with a few-micron-thick tear film, called the pre-lens tear film, on top of the lens (see Fig. 1).

There are a number of theoretical models in the literature for the suction pressure behind the contact lens. Reference 4 was the first, to our knowledge, to develop a model to predict the pressure distribution needed to conform an axisymmetric lens with an axisymmetric substrate. In their work, the contact lens is modeled as a linearly elastic membrane with variable thickness. Reference 5 extended the work of Ref. 4 to allow for gaps between the conformed contact lens and the eye. Finally, Ref. 6 considered the evolution of the suction pressure by linking the motion of the lens to the motion of the tear film. In addition, the pressure distribution has been measured through in vitro experiments, in which a soft contact lens is fitted to an axisymmetric rigid eye model.  

We present a new approach to compute the suction pressure. We argue that the important mechanical property of the contact lens for producing suction pressure is stretching, i.e., elastic tension. Also, we assume the contact lens must conform to the shape of the eye. This perspective allows us to derive a terse system of ordinary differential equations (ODEs) to determine the suction pressure under the contact lens. We solve this system numerically for various eye shapes and contact lens shapes to probe how the different contact lens design parameters, such as shape, thickness, and material properties, influence the suction pressure.

2. Formulation

We begin by presenting three short calculations illustrating that we can ignore bending forces, tangential forces, and the post-lens tear flow in the mathematical model for the suction pressure.

2.1. Bending forces are negligible

Bending forces, which of course dominate some elasticity problems, are insignificant in this problem; they contribute very little to the suction pressure in the post-lens
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tear film. To see this we consider, in the interest of computational simplicity, a radially symmetric contact lens of constant thickness \( \tau \), and whose shape is the graph of the fourth-degree polynomial function

\[
z = H \frac{r^4 - r_{\text{edge}}^4}{r_{\text{edge}}^4},
\]

where \( r \) is the distance from the center of the lens and \( r_{\text{edge}} \) is the radius of the base of the undeformed lens. With \( r_{\text{edge}} = 0.7 \text{ cm} \) and the constant \( H = 0.35 \text{ cm} \), this shape approximates a standard contact lens of diameter 1.4 cm.\(^{5,10}\) In order to deform this lens halfway to flat — that is, so that its shape became one-half the graph of Eq. (2.1) — would require a uniform pressure of

\[
P = \frac{E \tau^3}{12(1 - \sigma^2)} \frac{56H}{r_{\text{edge}}^4},
\]

where \( E \) is the Young’s modulus, and \( \sigma \) is the Poisson’s ratio. Here, \( E \tau^3/12(1 - \sigma^2) \) is the flexural rigidity of the lens, and \( -56H/r_{\text{edge}}^4 \) is the bilaplacian of the function \( z \) defined in Eq. (2.1).\(^{11}\) We find that the uniform pressure required for this deformation of a typical commercial contact lens, using values in Table 1, is under 12 dynes/cm\(^2\). Suction pressures produced by stretching stresses, examples of which we present in Sec. 3, are on the order of hundreds to thousands of dynes/cm\(^2\); the pressures induced by bending forces are small enough to be neglected in this basic approximation.

### 2.2. Tangential forces balance instantaneously

The stretching stresses of the contact lens can push/pull on the post-lens tear film (generate a suction pressure) and also drag the post-lens tear film around. Next, we show when the contact lens is pressed onto the eye and left to equilibrate, the tangential forces balance instantaneously; that is, we can neglect the dragging of the post-lens tear film.

Suppose we have a rectangular slice of a contact lens of length \( L \), width \( w \), and thickness \( \tau \) placed upon a tear film of thickness \( h \). At time \( t = 0 \), the lens is stretched by an amount \( \Delta L_0 \). If \( E \) is the Young’s modulus of the lens and \( \mu \) is the viscosity of the tear film, we can estimate the time it takes for the lens to relax back

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{\text{edge}} )</td>
<td>Radius of the undeformed contact lens</td>
<td>0.7 cm</td>
</tr>
<tr>
<td>( E )</td>
<td>Young’s modulus of the contact lens</td>
<td>( 10^7 \text{ dynes/cm}^2 )</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Thickness of the undeformed contact lens</td>
<td>( 5 \times 10^{-3} \text{ cm} )</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Poisson’s ratio of the contact lens</td>
<td>0.5</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Viscosity of the post-lens tear film</td>
<td>( 7 \times 10^{-3} \text{ g/cm s}^{-1} )</td>
</tr>
<tr>
<td>( h )</td>
<td>Thickness of the post-lens tear film</td>
<td>( 3 \times 10^{-4} \text{ cm} )</td>
</tr>
</tbody>
</table>
to its unstretched state. Let $\Delta L(t)$ be the amount by which the lens is stretched at time $t$. Then the elastic restoring force is $E w \tau \frac{\Delta L(t)}{L}$, and the viscous drag is $(\mu L/h)(d\Delta L/dt)$. When these forces balance, we have

$$E w \tau \frac{\Delta L(t)}{L} = \frac{\mu}{h} \frac{d\Delta L}{dt},$$

so $\Delta L(t) = \Delta L_0 \exp\left(\frac{-\tau E w h}{\mu L^2} t\right)$. \hspace{1cm} (2.3)

If we assume parameter values in Table 1, with $L = 1.4$ cm (diameter of the undeformed contact lens) and for simplicity, we assume $w = 1$ cm, then the relaxation time is on the order of a fraction of a millisecond. This implies that the contact lens would snap back instantaneously if stretched and placed on a thin film of tears (basically water). Therefore, the tangential force balance resolves itself immediately; that is, we do not have to resolve how the contact lens drags around the post-lens tear film.

2.3. Flow of post-lens tear film is negligible

Our basic approximation quantifies how the stretching stresses induce a suction pressure, which pushes and pulls on both the post-lens tear film and the eye. The final issue that we need to address to justify our approach is to show we can neglect the flow of the post-lens tear film in response to the suction pressure gradients that are established in it. A standard calculation shows that a pressure gradient $P/L$ pumps fluid with viscosity $\mu$ between two plates separated by distance $h$ at an average rate of $h^2 P/12 L \mu$.\hspace{1cm} (2.4) If we use parameter values in Table 1, with $L = 0.7$ cm (radius of the contact lens) and $P = 5000$ dynes/cm$^2$ (order of the suction pressures calculated in Sec. 3), then the velocity of the post-lens tear film is $8 \times 10^{-3}$ cm/s. Between blinks, which occur on an average of every 5 s, the post-lens tear fluid moves $4 \times 10^{-2}$ cm. The radius of the undeformed contact lens is 0.7 cm, so the post-lens tear film at the edge of the lens could penetrate only $0.04/0.7 = 6\%$; that is, at most 6% of post-lens tear fluid is replenished under the contact lens. Therefore, we neglect the flow of the post-lens tear film.

2.4. Derivation of suction pressure model

To quantify the elastic tension, we consider a radially symmetric contact lens on a radially symmetric eye. We let the unstressed shape of the lens be the graph of the function $z = g(r)$, where $r$ is the radial distance from the center of the lens. The lens is deformed so that it conforms to the shape of the eye, which is the graph of the function $z = f(R)$, where $R$ is the radial distance from the center of the corneal surface. The radial displacement of the lens in its deformed condition is given by a function $R(r)$.

The radial strain of the lens at rest position $r$ is

$$\frac{dR}{dr} \sqrt{1 + \left(\frac{f'(R)}{f'(r)}\right)^2} - \sqrt{1 + \left(\frac{g'(r)}{g'(r)}\right)^2} \over \sqrt{1 + g'(r)^2}.$$ \hspace{1cm} (2.4)
This is the fractional increase in the arc length of the lens in the radial direction at the rest position $r$. There is another strain associated with the radially symmetric deformation of the lens, the hoop strain:

$$\frac{R-r}{r}.$$

(2.5)

This is the fractional increase in length that the circle of lens points at undeformed coordinate $r$ experience as a consequence of their being deformed into a circle of radius $R(r)$. We assume that the lens is so thin that these strains are constant throughout the thickness of the lens; they vary only as functions of radial displacement from the center of the lens.

We regard the lens as a linear elastic material and we characterize it by its Young’s modulus, $E$, and its Poisson’s ratio, $\sigma$. Given the strains, we have the associated stresses:

$$\sigma_{rr} = \frac{E}{1 - \sigma^2} \left( \frac{\frac{dR}{dr} \sqrt{1 + f'(R)^2} - \sqrt{1 + g'(r)^2}}{\sqrt{1 + g'(r)^2}} + \sigma \frac{R-r}{r} \right),$$

(2.6)

$$\sigma_{\theta\theta} = \frac{E}{1 - \sigma^2} \left( \frac{R-r}{r} + \sigma \frac{dR}{dr} \sqrt{1 + f'(R)^2} - \sqrt{1 + g'(r)^2}}{\sqrt{1 + g'(r)^2}} \right).$$

(2.7)

The lens is thin and, in our approximation, the strains and stresses do not vary through its thickness, so we can simplify the formulation of our equations by integrating the stresses over the thickness of the lens to obtain tensions. We let $\tau(r)$ be the thickness of the lens at undeformed radial coordinate $r$. Then the tensions are

$$S_{rr} = \tau(r)\sigma_{rr} \quad \text{and} \quad S_{\theta\theta} = \tau(r)\sigma_{\theta\theta}.$$  

(2.8)

Note that these tensions have units of force per unit length, like a surface tension.

When the lens is pressed onto the eye and left to equilibrate, tangential forces balance instantaneously, according to our argument above. Pressure in the post-lens tear film develops and provides a normal force that balances the elastic forces normal to the surface. In order to derive equations from which we can compute this pressure — the suction pressure — and the elastic tensions, we identify two force balances.

First we consider the forces in the $z$-direction, the direction normal to the lens at its center, which we will call the vertical direction. Consider the part of the lens that is inside radial coordinate $R$. Two forces act on it in the $z$-direction: the vertical component of the pressure, which acts on the underside area of this part of the lens; and the radial tension, which acts tangential to this part of the lens along its circumference, and whose net component in the $z$-direction balances the pressure force:

$$2\pi RS_{rr} \frac{f'(R)}{\sqrt{1 + f'(R)^2}} + 2\pi \int_0^R \xi P(\xi)d\xi = 0.$$  

(2.9)
Note that by differentiating both sides of this equation and solving for $P$ we obtain an expression for the pressure in terms of the radial tension:

$$P(R) = -\frac{1}{R} \frac{d}{dR} \left( \frac{RS_{rr} f'(R)}{\sqrt{1 + f'(R)^2}} \right).$$

(2.10)

Next, we consider the balance of forces in the horizontal direction. We consider the same part of the lens but we cut it in half with a plane that contains the $z$-axis. The angular tension $S_{\theta\theta}$ produces a horizontal force by acting along the edge along which we halved the piece of the lens. The horizontal component of the radial tension acts on the other remaining edge of the piece of the lens. And the horizontal component of the pressure acts on the underside of the part of the lens:

$$\frac{RS_{rr}}{\sqrt{1 + f'(R)^2}} + \int_0^R (f(\xi) - f(R)) P(\xi) d\xi - \int_0^R S_{\theta\theta} \sqrt{1 + f'(\xi)^2} d\xi = 0. \quad (2.11)$$

With some differentiation and some algebraic manipulation we can eliminate $P$ and use $S_{\theta\theta} = \sigma S_{rr} + E \tau(r) \frac{R - r}{r}$ (which we derived from Eqs. (2.6) through (2.8)) to express $S_{\theta\theta}$ in terms of $S_{rr}$ to obtain a second-order ODE for $S_{rr}$:

$$\frac{d}{dR} \left( \frac{1}{f'(R)} \frac{d}{dR} \left( \frac{RS_{rr}}{\sqrt{1 + f'(R)^2}} \right) \right) + \frac{1}{R} \frac{d}{dR} \left( \frac{RS_{rr} f'(R)}{\sqrt{1 + f'(R)^2}} \right)$$

$$- \frac{d}{dR} \left( \left( \sigma S_{rr} + E \tau(r) \frac{R - r}{r} \right) \frac{\sqrt{1 + f'(R)^2}}{f'(R)} \right) = 0. \quad (2.12)$$

Radial symmetry implies that the functions $S_{rr}, S_{\theta\theta}, g(r), f(R), r(R)$, and $\tau(r)$ are even functions of their arguments. If we integrate the preceding equation from some small positive number $\epsilon$ to $R$, we obtain

$$\frac{1}{f'(R)} \frac{d}{dR} \left( \frac{RS_{rr}}{\sqrt{1 + f'(R)^2}} \right) + \frac{S_{rr} f'(R)}{f'(R)}$$

$$+ \int_{\epsilon}^R \frac{S_{rr} f'(\xi)}{\xi \sqrt{1 + f'(\xi)^2}} d\xi - \left( \sigma S_{rr} + E \tau(r) \frac{R - r}{r} \right) \frac{\sqrt{1 + f'(R)^2}}{f'(R)} = O(\epsilon).$$

So, by letting $\epsilon \to 0$, we obtain

$$\frac{1}{f'(R)} \frac{d}{dR} \left( \frac{RS_{rr}}{\sqrt{1 + f'(R)^2}} \right) + \frac{S_{rr} f'(R)}{f'(R)^2}$$

$$+ \int_0^R \frac{S_{rr} f'(\xi)}{\xi \sqrt{1 + f'(\xi)^2}} d\xi - \left( \sigma S_{rr} + E \tau(r) \frac{R - r}{r} \right) \frac{\sqrt{1 + f'(R)^2}}{f'(R)} = 0. \quad (2.13)$$

By introducing the notation $w = \int_0^R \frac{S_{rr} f'(\xi)}{\xi \sqrt{1 + f'(\xi)^2}} d\xi$, and by regarding Eq. (2.8) as an ODE for $r(R)$, we obtain a system of three first-order ODEs in three
unknowns,
\[ \frac{d}{dR} \left( \frac{RS_{rr}}{\sqrt{1+f'(R)^2}} \right) = -f'(R)w - \frac{S_{rr}f'(R)^2}{\sqrt{1+f'(R)^2}} \]
\[ + \left( \sigma S_{rr} + E\tau(r)\frac{R-r}{r} \right) \sqrt{1+f'(R)^2}, \]
\[ \frac{dw}{dR} = \frac{S_{rr}f'(R)}{R\sqrt{1+f'(R)^2}}, \]
\[ \frac{dr}{dR} = \frac{\sqrt{1+f'(R)^2}}{\sqrt{1+g'(r)^2}}((1-\sigma^2)\frac{S_{rr}}{E\tau(r)} + 1 - \sigma \frac{R-r}{r}). \]

We supplement these equations with initial values for the unknown functions:
\[ S_{rr}(0) = S_0, \quad w(0) = 0, \quad \text{and} \quad r(0) = 0. \] (2.15)
The initial values for \( w \) and \( r \) follow directly from their definitions. We need a criterion for determining \( S_0 \), the radial tension at the center of the lens. This criterion comes from a physical condition at the edge of the lens, not one at the center. At the edge of the lens there is no applied force, so the radial tension must be zero:
\[ S_{rr}(R_{\text{edge}}) = 0. \] (2.16)

We do not know \( R_{\text{edge}} \) without solving the equations. What we do know is that \( r(R_{\text{edge}}) = r_{\text{edge}} \), that is, we know the outer radius of the undeformed lens.

Equations (2.14) – (2.16) constitute a problem for determining the unknown functions \( S_{rr}(R), w(R), \) and \( r(R) \) and the unknown constant \( S_0 \). The system has several notable features. It is singular at its initial point, \( R = 0 \), because the coefficient of \( \frac{dS_{rr}}{dR} \) vanishes there. It is nonlinear. And the condition (2.16) makes it a non-standard boundary value problem. This system cannot be solved in closed form, we must solve it numerically.

The issue of well posedness of the system (2.14)-(2.16) is not a simple one; that it is well posed is not obvious. The physical reasoning that we have presented suggests strongly that a solution of the problem exists in general. It is less clear that there is a unique solution of this system in all cases, though it seems reasonable to expect that if \( g(r) \) and \( f(R) \) are sufficiently tame functions, for example if neither of their graphs has inflection points, then the solution is unique.

The condition (2.16) requires that we use a shooting method to solve the system numerically. We use a packaged ODE solver \texttt{ode23} in MATLAB to solve Eq. (2.14) as an initial value problem for any given value of \( S_0 \). We integrate from \( R = 0 \) to the value of \( R \) at which \( r(R) = r_{\text{edge}} \), and we call that value \( R_{\text{edge}} \). If condition (2.16) is satisfied, then we have found the solution; otherwise, we integrate again with a different value of \( S_0 \). Therefore, we regard \( S_{rr}, w, \) and \( r \) as functions of both \( R \) and \( S_0 \), so our integration yields \( S_{rr}(R, S_0) \).
3. Results

We begin by examining the suction pressure under a contact lens of constant thickness, $\tau(r) = \tau_0$. Unless otherwise stated, we take the Young’s modulus to be $E = 10^7$ dynes/cm$^2$, the characteristic thickness of the contact lens to be $\tau_0 = 5 \times 10^{-3}$ cm, and the Poisson’s ratio of the contact lens to be $\sigma = 0.5$ (that is, we take the lens to be incompressible).

To understand the results, we approximate the contact lens by a series of rings with radius $r$, width $\sqrt{1 + g'(r)^2} dr$, and thickness $\tau(r)$ nested inside each other (see Fig. 2). Then the radial strain characterizes either a widening of the ring, if it is positive, or a thinning of the ring, if it is negative, whereas the hoop strain describes either the lengthening of the circumference of the ring, if it is positive, or the shortening of the circumference of the ring, if it is negative. As a consequence of the strains, the contact lens is either under radial tension (positive radial tension — the ring gets wider) or radial compression (negative radial tension — the ring gets narrower) as well as angular tension (positive angular tension — the circumference of the ring gets longer) or angular compression (negative angular tension — the circumference of the rings gets shorter).

We first consider the case in which the undeformed shape of the contact lens, the graph of $z = g(r)$, and the shape of the eye, the graph of $z = f(R)$, are spherical caps of the form

$$C(r) = \sqrt{\left(\frac{h_0^2 + a_0^2}{2h_0}\right)^2 - r^2} - \frac{a_0^2 - h_0^2}{2h_0},$$

where $h_0$ is the height of the cap, $a_0$ is the radius of the base of the cap, and the radius of curvature is $R = (h_0^2 + a_0^2)/(2h_0)$; note all quantities are measured in cm.

Figure 3 shows the numerical results in the case in which the shape of the eye ($a_0 = 3/2, h_0 = 1/2, R = 5/2$) has a smaller mean curvature than the undeformed shape of the contact lens ($a_0 = 1, h_0 = 1/2, R = 5/4$). For the contact lens to conform to an eye with less curvature, the contact lens must narrow each ring, (negative

![Fig. 2. An illustration of the contact lens approximated by a series of rings with a specified radius, width, and thickness.](image)
radial strain), shorten the circumference of the rings near the top of the lens (negative hoop strain) and lengthen the circumference of the rings near the edge of the lens (positive hoop strain). Therefore, the contact lens is under radial compression. This results in a suction pressure, as shown in Fig. 3(e), that is negative in the center of the lens and positive at the edge of the lens. Figure 3(f) shows how the radial coordinates of material points in the undeformed contact lens \( r \) are mapped to the coordinates of material points in the deformed contact lens \( R \). Specifically, the solid line plots \( y = R \), while the dashed line plots \( y = r(R) \), so for a given value of \( R \), the distance between the solid and dashed lines represents the radial displacement of the contact lens.

Fig. 3. The results for a spherical-cap-shaped eye \( (a_0 = 3/2, h_0 = 1/2, \mathcal{R} = 5/2) \) having less mean curvature than the undeformed spherical-cap-shaped contact lens \( (a_0 = 1, h_0 = 1/2, \mathcal{R} = 5/4) \).
Next, we switch the curvatures so that the shape of the eye ($a_0 = 1$, $h_0 = 1/2$, $R = 5/4$) has more mean curvature than the undeformed shape of the contact lens ($a_0 = 3/2$, $h_0 = 1/2$, $R = 5/2$). For the contact lens to conform to an eye with more curvature, the contact lens must widen each ring (positive radial strain) lengthen the circumference of the rings near the top of the lens (positive hoop strain) and shorten the circumference of the rings near the edge of the lens (negative hoop strain). Therefore, the contact lens is under radial expansion. This results in a suction pressure, shown in Fig. 4(e), that is positive in the center of the lens and positive at the edge of the lens.

Fig. 4. The results for a spherical-cap-shaped eye ($a_0 = 1$, $h_0 = 1/2$, $R = 5/4$) having more mean curvature than the undeformed spherical-cap-shaped contact lens ($a_0 = 3/2$, $h_0 = 1/2$, $R = 5/2$).
To summarize, a flatter eye (relative to the undeformed lens) produces a more negative pressure in the center of the contact lens and tends to draw fluid in under the lens, while a pointed eye creates a more positive pressure under the center of the lens and tends to push tear fluid out from under the lens. If the eye were flat (a horizontal plane) then regardless of the shape of the undeformed contact lens a zero pressure would be obtained. When the contact lens conforms to a flat eye, there is no radial tension acting in the $z$-direction and therefore no pressure is needed to balance this force. The flat deformed contact lens is an unstable equilibrium solution.

We now apply our model to typical commercial contact lenses and realistic eye shapes. To good approximation, the undeformed contact lens is a spherical cap. A typical radius is given by $r_{\text{edge}} = 0.7 \text{ cm}$. We consider lenses with three different radii of curvature of $R = 0.84, 0.87, \text{ and } 0.9 \text{ cm}$. The thickness of the lens is initially set to be constant with $\tau_0 = 5 \times 10^{-3} \text{ cm}$. The surface of the eye is composed of three different regions called the cornea (which contains the pupil), the limbus (which is a transition region), and the sclera (which is the white part of the eye). Cross-sectional measurements indicate that a good approximation of the cornea is the sharp end of an ellipse whose equation is

$$\left(\frac{r}{0.87}\right)^2 + \left(\frac{z}{0.96}\right)^2 = 1, \quad (3.2)$$

where $r$ and $z$ are measured in cm. The diameter of the cornea is $1.2 \text{ cm}$. The sclera has less mean curvature than the cornea. We approximate the sclera with a spherical cap whose base radius is $a_0 = 0.75 \text{ cm}$ and radius of curvature $R = 1.2 \text{ cm}$. For simplicity, we ignore the limbus region and therefore the surface of our idealized eye is not smooth.

Figure 5 shows the results associated with a contact lens whose radius of curvature is $R = 0.87 \text{ cm}$. On the cornea, the contact lens must lengthen the circumference of the rings near the center (positive hoop strain), and shorten the circumference of its rings near the edge of the cornea (negative hoop strain) to conform. Whereas, to conform to the sclera, which has a different radius of curvature, the contact lens must lengthen the circumference of the rings near the edge (positive hoop strain), and widen its rings (positive radial strain). As a consequence, the contact lens is under positive radial tension. The suction pressure is positive in the center and at the edge, and negative near the transition region. If the contact lens were a spherical drop of water, then the Laplace pressure inside the drop would be $2\gamma/R = 161 \text{ dynes/cm}^2$, where $\gamma$ denotes surface tension. The suction pressure generated by contact lens can be up to 13 times the Laplace pressure.

The radius of curvature of the contact lens influences the suction pressure above the cornea as well as the maximum value of the suction pressure at the edge of the contact lens. In Fig. 6(a), we plot the suction pressure for the lenses for different radii of curvature. As the radius of curvature increases, the suction pressure at the center of the cornea increases, whereas the pressure at the edge of the lens decreases.
Therefore, there are potential tradeoffs. We note that a positive suction pressure would push on the surface of the eye.

One contact lens design parameter varied by the industry is the thickness of the lens. In Fig. 6(b), we compare the suction pressures of two lenses. The first has a radius of curvature of \( R = 0.87 \text{ cm} \) and a thickness \( \tau = \tau_0 \) (solid line) and the second has a radius of curvature of \( R = 0.87 \text{ cm} \) and a varying thickness of \( \tau = \tau_0 (1 - (r/r_{\text{edge}})^2) \) (dashed line). As the lens becomes thinner at the edge, the suction pressure decreases significantly. In fact, the magnitude of suction pressure in this case is less than the Laplace pressure inside a spherical droplet of water with radius 0.87 cm. From a perspective of protecting the ocular surface, the thinning of the lens seems advantageous.
Finally, soft contact lenses are made from hydrogels. Therefore, the contact lens may have a Poisson’s ratio less than 0.5. We found on the realistic eye shape the suction pressure is not significantly influenced by the Poisson’s ratio.

4. Conclusion

We presented a model to characterize the suction pressure under the contact lens. The system of ODEs captures the basic physics — the elastic tension — needed to begin to understand a “working” contact lens. Our numerical results indicate for a fixed eye shape that the suction pressure at the center of the lens increases in magnitude as the radius of curvature is increased, whereas the positive suction pressure at the edge decreases; the negative pressure generated in the transition region also increases in magnitude with increasing radii of curvature. In addition, we found thinner contact lenses to create suction pressures whose magnitudes are smaller. Therefore, we can begin to create “rules of thumb” for contact lens design.

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