**T312-03-03** A wad of sticky clay having mass \( m \) is fired with the velocity represented in Figure 03-03 toward a solid uniform cylinder having mass \( M \) and radius \( R \). The cylinder is initially at rest and is mounted on a fixed horizontal axle that runs through its center of mass as shown. The line of motion of the wad of clay is perpendicular to the axle and at a distance \( d < R \) from the center of the cylinder. (a) Find the angular speed of the system just after the clay strikes and sticks to the cylinder. (b) Is the mechanical energy of the clay-cylinder system conserved in the process? Explain your answer. (c) Is the linear momentum of the clay-cylinder system conserved in the process? Explain your answer.

![Figure 03-03](image)

\( \mathbf{v}_i \)

\( d \)

\( R \)

\( M \)

\( m \)

(a) **Internal torques only \( \Rightarrow \) use conservation of angular momentum**

\[ L_i = L_f \]  

(let into the page be positive)

\[ L_i = L_{i\,\text{clay}} + L_{i\,\text{cyl}} \]

\[ L_i = m\mathbf{r}_i \times \mathbf{v}_i = m\mathbf{v}_i d \]  

Clay as point mass

\[ L_f = I_f \omega_f \]

\[ I_f = I_{cyl} + mR^2 \]

\[ = \left( \frac{1}{2}MR^2 + mR^2 \right) \omega_f \]
So

\[ MV_i d = \left( \frac{1}{2} MR_i^2 + mR_i^2 \right) \omega_f \]

\[ \omega_f = \left( \frac{m}{2M+m} \right) \nu_i \frac{d}{R^2} \]

(b) No

\[ KE_i = \frac{1}{2} m v_i^2 \]

\[ KE_f = \frac{1}{2} \int_0^\infty \omega_f^2 = \frac{1}{2} \left( \frac{1}{2} MR_i^2 + mR_i^2 \right) \int_0^\infty \left( \frac{m}{2M+m} \right) \nu_i^2 \frac{d}{R^2}^2 \]

\[ = \frac{1}{2} \left( \frac{1}{2} MR_i^2 \right) R^2 \frac{m^2}{(2M+m)^2} \nu_i^2 \frac{d^2}{R^2} \]

\[ = \left( \frac{m}{2M+m} \right) \left( \frac{d}{R} \right)^2 \left( \frac{1}{2} m v^2 \right) \]

\[ KE_f = \left( \frac{m}{2M+m} \right) \frac{d^2}{R^2} KE_i \]

\[ \left( \frac{m}{2M+m} \right) \frac{d^2}{R^2} < 1 \Rightarrow KE_f < KE_i \]
(c) No

Before the collision $\vec{P}_{\text{sys}}$ is purely horizontal.

Just after the collision it has a vertical component

(and $P_{fx} \neq P_{ix}$ either)